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不动点方法与概率时滞脉冲模糊系统的稳定性^①

黄家琳¹, 李兴贵²

1. 四川三河职业学院 基础部, 四川 泸州 646200; 2. 成都师范学院 数学学院, 成都 611130

摘要: 通过在完备距离空间上定义压缩映射, 利用 T-S 模糊规则、概率时滞的相关性质和压缩映像原理导出了一类 T-S 模糊概率时滞脉冲双向联想记忆神经网络的稳定性的代数判据. 特点在于, 导出系统解的存在性的同时给出了该解的稳定性结论. 最后, 数值实例证实了所述方法的有效性.

关键词: 双向联想记忆神经网络; 概率时滞; 不动点定理

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以前很多文献用李雅普诺夫函数法导出神经网络的稳定性^[1-9], 然而每一种方法有其局限性, 不动点方法是李雅普诺夫函数法的替代方法之一^[10-16]. 本文作者考虑用压缩映射原理结合线性矩阵不等式方法给出一类 T-S 模糊概率时滞脉冲双向联想记忆神经网络的稳定性的代数判据(LMI). 特别地, LMI 判据适合于计算机 Matlab LMI 工具箱编程运算, 符合实际工程中大型计算的要求. 由于方法和条件的不同, 本文更新了相关文献^[14-16]的结果.

1 预备知识

本文考虑下述由 IF-THEN 规则所描述的 T-S 模糊双向联想记忆神经网络系统模型:

模糊规则 j 令 $t \in [0, +\infty)$, $t \neq t_k$, $k = 1, 2, \dots$, 若 $\omega_1(t) = \mu_{j1}, \dots, \omega_m(t) = \mu_{jm}$, 则

$$\begin{cases} \frac{dx(t)}{dt} = -\mathbf{A}x(t) + \mathbf{C}_j f(y(t - \tau(t))) \\ \frac{dy(t)}{dt} = -\mathbf{B}y(t) + \mathbf{D}_j g(x(t - h(t))) \\ x(t_k^+) - x(t_k) = \rho(x(t_k)), y(t_k^+) - y(t_k) = \rho(y(t_k)) \\ x(s) = \xi(s), y(s) = \eta(s) \quad s \in [-\tau, 0] \end{cases} \quad (1)$$

其中

$$\begin{aligned} x(t) &= (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n \\ y(t) &= (y_1(t), y_2(t), \dots, y_n(t))^T \in \mathbb{R}^n \\ \xi(s), \eta(s) &\in C([-\tau, 0], \mathbb{R}^n) \end{aligned}$$

$\{\mu_{jk}\}$ 是模糊集 ($j = 1, 2, \dots, J$; $k = 1, 2, \dots, m$), $\omega_k(t)$ 是前件变量, m 为前件变量的个数, J 是 IF-THEN

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作者简介: 黄家琳(1949-), 男, 教授, 主要从事微分方程理论及其应用的研究.

通信作者: 李兴贵, 教授, 博士.

规则的个数. 激活函数

$$f(x(t - \tau(t))) = (f_1(x_1(t - \tau(t))), f_2(x_2(t - \tau(t))), \dots, f_n(x_n(t - \tau(t))))^T \in \mathbb{R}^n$$

$$g(x(t - h(t))) = (g_1(x_1(t - h(t))), g_2(x_2(t - h(t))), \dots, g_n(x_n(t - h(t))))^T \in \mathbb{R}^n$$

脉冲函数

$$\rho(x(t)) = (\rho_1(x_1(t)), \rho_2(x_2(t)), \dots, \rho_n(x_n(t)))^T \in \mathbb{R}^n$$

时滞 $0 \leq \tau(t), h(t) \leq \tau, \forall i = 1, 2, \dots, n$. 我们简记时滞神经元之间相互联络的权系数矩阵 $\mathbf{C}_j, \mathbf{D}_j$ 是 n 维方阵. 脉冲时刻 $t_k (k = 1, 2, \dots)$ 满足 $0 < t_1 < t_2 < \dots, \lim_{k \rightarrow \infty} t_k = \infty$. $x(t_k^+)$ 和 $x(t_k^-)$ 分别表示 $x(t)$ 在 t_k 时刻的右极限和左极限. 假设 $x(t_k^-) = x(t_k) (\forall k = 1, 2, \dots)$. 令 $t \geq 0, t \neq t_k, k = 1, 2, \dots$, 由单点模糊化、乘积推理和平均加权反模糊化得到模糊系统的整个状态方程为

$$\begin{cases} \frac{dx(t)}{dt} = -\mathbf{A}x(t) + \sum_{j=1}^J \rho_j(\omega(t)) \mathbf{C}_j f(y(t - \tau(t))) \\ \frac{dy(t)}{dt} = -\mathbf{B}y(t) + \sum_{j=1}^J \rho_j(\omega(t)) \mathbf{D}_j g(x(t - h(t))) \\ x(t_k^+) - x(t_k) = \rho(x(t_k)), y(t_k^+) - y(t_k) = \rho(y(t_k)) \\ x(s) = \xi(s), y(s) = \eta(s) \quad s \in [-\tau, 0] \end{cases} \quad (2)$$

其中

$$\omega(t) = (\omega_1(t), \dots, \omega_m(t)) \quad \rho_j(\omega(t)) = \frac{\mathbf{r}_j(\omega(t))}{\sum_{i=1}^J \mathbf{r}_i(\omega(t))}$$

$\mathbf{r}_j(\omega(t))$ 为相应于规则 j 的隶属度函数, 且 $\sum_{j=1}^J \rho_j(\omega(t)) = 1, \rho_j(\omega(t)) \geq 0$.

由于实际系统中的时滞达到较大值的概率很小, 于是我们需要考虑概率时滞

$$\mathbb{P}(0 \leq \tau(t) \leq \tau_1) = c_0 \quad \mathbb{P}(\tau_1 < \tau(t) \leq \tau_2) = 1 - c_0$$

设实数 $c_0 \leq 1$, 定义随机变量

$$\mathcal{C}(t) = \begin{cases} 1 & 0 \leq \tau(t) \leq \tau_1 \\ 0 & \tau_1 < \tau(t) \leq \tau_2 \end{cases}$$

令 $t \geq 0, t \neq t_k, k = 1, 2, \dots$, 考虑概率时滞模糊系统

$$\begin{cases} \frac{dx(t)}{dt} = -\mathbf{A}x(t) + \sum_{j=1}^J \rho_j(\omega(t)) [c_0 \mathbf{C}_j f(y(t - \tau_1(t), x)) + (1 - c_0) \mathbf{C}_j f(y(t - \tau_2(t), x)) + \\ (\mathcal{C} - c_0)(\mathbf{C}_j f(y(t - \tau_1(t), x)) - \mathbf{C}_j f(y(t - \tau_2(t), x)))] \\ \frac{dy(t)}{dt} = -\mathbf{B}y(t) + \sum_{j=1}^J \rho_j(\omega(t)) [c_0 \mathbf{D}_j g(x(t - h_1(t), x)) + (1 - c_0) \mathbf{D}_j g(x(t - h_2(t), x)) + \\ (\mathcal{C} - c_0)(\mathbf{D}_j g(x(t - h_1(t), x)) - \mathbf{D}_j g(x(t - h_2(t), x)))] \\ x(t_k^+) - x(t_k) = \rho(x(t_k)), y(t_k^+) - y(t_k) = \rho(y(t_k)) \\ x(s) = \xi(s), y(s) = \eta(s) \quad s \in [-\tau, 0] \end{cases} \quad (3)$$

本文假设: $f(0) = g(0) = \rho(0) = 0 \in \mathbb{R}^n$; 对角矩阵 $\mathbf{A} = \text{diag}(a_1, a_2, \dots, a_n), \mathbf{B} = \text{diag}(b_1, b_2, \dots, b_n)$

正定; 对角矩阵 $\mathbf{F}, \mathbf{G}, \mathbf{H}$ 分别是向量函数 f, g, ρ 的利普希茨常数矩阵.

2 主要结论与证明

定理 1 假设存在常数 $0 < \lambda < 1$, 使得

$$\begin{cases} \sum_{j=1}^J |C_j| F + \frac{1}{\delta} H + AH - \lambda A < 0 \\ \sum_{j=1}^J |D_j| G + \frac{1}{\delta} H + BH - \lambda B < 0 \end{cases} \quad (4)$$

则脉冲时滞系统(2) 是指数稳定的, 其中 $\delta = \inf_{k=1,2,\dots} (t_{k+1} - t_k) > 0$.

证 首先定义空间 $\Omega = \Omega_1 \times \Omega_2$.

设 $\Omega_i (i=1,2)$ 是这样的函数空间, 其函数 $q_i(t): [-\tau, \infty) \rightarrow \mathbb{R}^n$ 满足以下 4 条:

- (a) $q_i(t)$ 连续于 $t \in [0, +\infty) \setminus \{t_k\}_{k=1}^\infty$;
- (b) $q_1(t) = \xi(t), q_2(t) = \eta(t) (\forall t \in [-\tau, 0])$;
- (c) $\lim_{t \rightarrow t_k^-} q_i(t) = q_i(t_k)$, 且 $\lim_{t \rightarrow t_k^+} q_i(t)$ 存在 ($\forall k=1,2,\dots$);
- (d) 当 $t \rightarrow \infty$ 时 $e^{\gamma t} q_i(t) \rightarrow 0$, 其中 $\gamma > 0$ 是常数, 满足 $\gamma < \min\{\lambda_{\min} A, \lambda_{\min} B\}$.

则易证 Ω 是下述度量下的完备空间:

$$\text{dist}(\bar{q}, \tilde{q}) = \max_{i=1,2,\dots,2n-1,2n} (\sup_{t \geq -\tau} |\bar{q}^{(i)}(t) - \tilde{q}^{(i)}(t)|)$$

其中

$$\bar{q} = \bar{q}(t) = \begin{pmatrix} \bar{q}_1(t) \\ \bar{q}_2(t) \end{pmatrix} = (\bar{q}^{(1)}(t), \bar{q}^{(2)}(t), \dots, \bar{q}^{(2n)}(t))^T \in \Omega$$

$$\tilde{q} = \tilde{q}(t) = \begin{pmatrix} \tilde{q}_1(t) \\ \tilde{q}_2(t) \end{pmatrix} = (\tilde{q}^{(1)}(t), \dots, \tilde{q}^{(2n)}(t))^T \in \Omega$$

这里 $\bar{q}_i \in \Omega_i, \tilde{q}_i \in \Omega_i, i=1,2$.

现定义压缩映射 $P: \Omega \rightarrow \Omega$, 这需 3 步来实现.

第一步, 关于系统(3), 我们可以构造如下映射:

$$\begin{cases} P(x(t)) = e^{-At} \xi(0) + e^{-At} \left[\int_0^t e^{As} \sum_{j=1}^J \rho_j(\omega(t)) C_j (\mathcal{C}f(y(s-\tau_1(s))) + (1-\mathcal{C})f(y(s-\tau_2(s)))) ds + \sum_{0 < t_k < t} e^{A t_k} \rho(x_{t_k}) \right] & t \geq 0 \\ P(y(t)) = e^{-Bt} \eta(0) + e^{-Bt} \left[\int_0^t e^{Bs} \sum_{j=1}^J \rho_j(\omega(t)) D_j (\mathcal{C}g(x(s-h_1(s))) + (1-\mathcal{C})g(x(s-h_2(s)))) ds + \sum_{0 < t_k < t} e^{B t_k} \rho(y_{t_k}) \right] & t \geq 0 \\ P \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \xi(t) \\ \eta(t) \end{pmatrix} & \forall t \in [-\tau, 0] \end{cases} \quad (5)$$

我们不难证明 P 是 Ω 上的压缩映射.

第二步, 不难证明 $P \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in \Omega, \forall \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \in \Omega$. 换言之, $P \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ 满足条件(a) - (d).

第三步, 证明(5) 式定义的 P 是压缩映射.

对任给 $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \in \Omega$, 我们有

$$\left| P \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} - P \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right| \leq$$

$$\begin{aligned}
& \left[e^{-A^t} \int_0^t e^{As} \sum_{j=1}^J |C_j| (|\mathcal{C}| |f(y(s-\tau_1(s))) - f(\bar{y}(s-\tau_1(s)))| + (1-|\mathcal{C}|) |f(y(s-\tau_2(s))) - f(\bar{y}(s-\tau_2(s)))|) ds \right. \\
& \left. e^{-B^t} \int_0^t e^{Bs} \sum_{j=1}^J |D_j| (|\mathcal{C}| |g(x(s-h_1(s))) - g(\bar{x}(s-h_1(s)))| + (1-|\mathcal{C}|) |g(x(s-h_2(s))) - g(\bar{x}(s-h_2(s)))|) ds \right] + \\
& \left(e^{-A^t} \sum_{0 < t_k < t} e^{A t_k} |\rho(x_{t_k}) - \rho(\bar{x}_{t_k})| \right) \\
& \left(e^{-B^t} \sum_{0 < t_k < t} e^{B t_k} |\rho(y_{t_k}) - \rho(\bar{y}_{t_k})| \right) \leq \\
& \left[\left(A^{-1} \sum_{j=1}^J |C_j| F \mu \right) + \frac{1}{\delta} \left(e^{-A^t} \left(\int_0^t e^{As} ds + \delta e^{At} \right) H \mu \right) \right] \text{dist} \left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right) \leq \\
& \left[\left(A^{-1} \sum_{j=1}^J |C_j| F + \frac{1}{\delta} A^{-1} H + H \right) \mu \right] \text{dist} \left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right) < \\
& \left[\left(B^{-1} \sum_{j=1}^J |D_j| G + \frac{1}{\delta} B^{-1} H + H \right) \mu \right] \text{dist} \left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right) < \\
& \lambda \begin{pmatrix} \mu \\ \mu \end{pmatrix} \text{dist} \left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right)
\end{aligned}$$

因此

$$\text{dist} \left(P \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, P \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right) \leq \lambda \text{dist} \left(\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \begin{pmatrix} \bar{x}(t) \\ \bar{y}(t) \end{pmatrix} \right)$$

所以 $P: \Omega \rightarrow \Omega$ 是压缩映射, 从而存在 P 在 Ω 上的不动点 $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$. 即 $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ 是模糊时滞脉冲系统(2)的

解, 满足 $e^{\gamma t} \left\| \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \right\| \rightarrow 0 (t \rightarrow +\infty)$. 证毕.

3 数值实例

例 1 考虑下列模糊 BAM 神经网络:

模糊规则 1

令 $t \geq 0, t \neq t_k, k = 1, 2, \dots$, 若 $\omega_1(t) = \frac{1}{e^{-5\omega_1(t)}}$, 则

$$\begin{cases} \frac{dx(t)}{dt} = -\mathbf{A}x(t) + \mathbf{C}_1 f(y(t-\tau(t))) \\ \frac{dy(t)}{dt} = -\mathbf{B}y(t) + \mathbf{D}_1 g(x(t-h(t))) \\ x(t_k^+) - x(t_k) = \rho(x(t_k)), y(t_k^+) - y(t_k) = \rho(y(t_k)) \\ x(s) = \xi(s), y(s) = \eta(s) \quad s \in [-\tau, 0] \end{cases} \quad (6)$$

模糊规则 2

令 $t \geq 0, t \neq t_k, k = 1, 2, \dots$, 若 $\omega_2(t) = 1 - \frac{1}{e^{-5\omega_1(t)}}$, 则

$$\begin{cases} \frac{dx(t)}{dt} = -\mathbf{A}x(t) + \mathbf{C}_2 f(y(t - \tau(t))) \\ \frac{dy(t)}{dt} = -\mathbf{B}y(t) + \mathbf{D}_2 g(x(t - h(t))) \\ x(t_k^+) - x(t_k) = \rho(x(t_k)), y(t_k^+) - y(t_k) = \rho(y(t_k)) \\ x(s) = \xi(s), y(s) = \eta(s) \quad s \in [-\tau, 0] \end{cases} \quad (7)$$

其中 $\tau(t) = h(t) = \tau = 0.8$, $t_1 = 0.3$, $t_k = t_{k-1} + 0.3k$, $\delta = 0.5$, $x(s) = \tanh s$, $y(s) = 2\sin s$, $f(x) = 0.1\sin x$, $g(x) = 0.09\sin x$, $\rho(x) = 0.1x$, $\mathbf{A} = (2)$, $\mathbf{B} = (1.95)$, $\mathbf{C}_1 = (0.02)$, $\mathbf{C}_2 = (0.03)$, $\mathbf{D}_1 = (0.15)$, $\mathbf{D}_2 = (0.18)$, $\mathbf{F} = (0.1)$, $\mathbf{G} = (0.09)$, $\mathbf{H} = (0.1)$.

利用计算机 Matlab LMI 工具箱解(4) 式, 得到可行性数据

$$\lambda = 0.8115$$

则由定理 1 知, 模糊系统(6)-(7) 是指数稳定的.

4 总 结

本文用不动点方法研究了模糊脉冲概率时滞 BAM 神经网络系统的稳定性, 其优点是利用压缩映像原理给出系统解的存在性的同时, 也给了该解的全局指数型稳定结论, 这点由本文函数空间的构造可以看出.

参考文献:

- [1] RAO R F. Global Stability of a Markovian Jumping Chaotic Financial System with Partially Unknown Transition Rates under Impulsive Control Involved in the Positive Interest Rate [J]. Mathematics, 2019, 7(7): 579.
- [2] LI H F, JIANG H J, HU C. Existence and Global Exponential Stability of Periodic Solution of Memristor-Based BAM Neural Networks with Time-Varying Delays [J]. Neural Networks, 2016, 75: 97-109.
- [3] RAO R F, ZHONG S M, PU Z L. On the Role of Diffusion Factors in Stability Analysis for p -Laplace Dynamical Equations Involved to BAM Cohen-Grossberg Neural Network [J]. Neurocomputing, 2017, 223: 54-62.
- [4] 饶若峰, 黄家琳, 钟守铭. 反应扩散 BAM 神经网络的全局指数稳定性 [J]. 吉林大学学报(理学版), 2012, 50(6): 1086-1090.
- [5] 俸 卫. T-S 模糊马尔可夫跳跃时滞 Cohen-Grossberg 神经网络的几乎必然指数稳定性 [J]. 西南师范大学学报(自然科学版), 2015, 40(5): 10-12.
- [6] 黄家琳, 饶若峰. Cohen-Grossberg 神经网络的全局指数稳定性 [J]. 西南大学学报(自然科学版), 2016, 38(2): 78-82.
- [7] RAO R F. Delay-Dependent Exponential Stability for Nonlinear Reaction-Diffusion Uncertain Cohen-Grossberg Neural Networks with Partially Known Transition Rates via Hardy-Poincare Inequality [J]. Chinese Annals of Mathematics(Series B), 2014, 35(4): 575-598.
- [8] RAO R F, ZHONG S M, WANG X R. Stochastic Stability Criteria with LMI Conditions for Markovian Jumping Impulsive BAM Neural Networks with Mode-Dependent Time-Varying Delays and Nonlinear Reaction-Diffusion [J]. Communications in Nonlinear Science and Numerical Simulation, 2014, 19(1): 258-273.
- [9] RAO R F, TANG G Z, GONG J Q, et al. Robust Stability of Nonlinear Diffusion Fuzzy Neural Networks with Parameter Uncertainties and Time Delays [J]. Complexity, 2018, 2018: 1-19.
- [10] RAO R F, ZHONG S M, PU Z L. Fixed Point and p -Stability of T-S Fuzzy Impulsive Reaction-Diffusion Dynamic Neural Networks with Distributed Delay Via Laplacian Semigroup [J]. Neurocomputing, 2019, 335: 170-184.
- [11] RAO R F, ZHONG S M. Stability Analysis of Impulsive Stochastic Reaction-Diffusion Cellular Neural Network with Distributed Delay via Fixed Point Theory [J]. Complexity, 2017, 2017: 1-9.
- [12] RAO R F, PU Z L, ZHONG S M, et al. Fixed Points and Exponential Stability for Impulsive Time-Delays BAM Neural Networks via LMI Approach and Contraction Mapping Principle [J]. Mathematical Problems in Engineering, 2016,

2016; 1-8.

- [13] RAO R F, PU Z L. LMI-Based Stability Criterion of Impulsive T-S Fuzzy Dynamic Equations Via Fixed Point Theory [J]. *Abstract and Applied Analysis*, 2013, 2013: 1-9.
- [14] 牟天伟, 饶若峰. 不动点原理在时滞 BAM 神经网络稳定性分析中的一个应用 [J]. *西南大学学报(自然科学版)*, 2017, 39(6): 5-9.
- [15] WANG X R, RAO R F, ZHONG S M. LMI-Based Stability Criterion for Impulsive CGNNs via Fixed Point Theory [J]. *Mathematical Problems in Engineering*, 2015, 2015: 1-10.
- [16] 李兴贵, 黄家琳. 不动点技巧在反应扩散模糊随机周期时滞系统稳定性分析中的应用 [J]. *西南大学学报(自然科学版)*, 2019, 41(6): 64-72.

The Fixed Point Theorem and the Stability of Fuzzy Impulsive Systems with Probability Time-Delays

HUANG Jia-lin¹, LI Xing-gui²

1. *Department of Elementary Education, Sichuan Sanhe College of Professionals, Luzhou Sichuan 646200, China;*

2. *Department of Mathematics, Chengdu Normal University, Chengdu 611130, China*

Abstract: By defining a contraction mapping on a complete distance space, the authors employ the T-S fuzzy rule, probabilistic time-delay property and contraction mapping principle to derive an algebraic criterion for the stability of a class of T-S fuzzy probabilistic time-delay impulsive Bidirectional Associative Memory neural networks. Remarkably, the stability of the solution is given as soon as the existence of the solution of the system is derived. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Key words: bi-directional associative memory neural networks; probability time-delay; fixed point theorem

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