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时间尺度上带有反应扩散项和狄利克雷边值的 BAM 神经网络模型的稳定性①

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摘要：在时间尺度上通过构造合适的李雅普诺夫函数，并使用一些分析技巧，研究带有反应扩散项和狄利克雷边值的 BAM 神经网络模型，获得了一些使得带有反应扩散项和狄利克雷边值的 BAM 神经网络模型的平衡点全局指数稳定的充分条件，并将以前的一些结论在时间尺度上做了扩展.

关　键　词：BAM 神经网络；狄利克雷边值；反应扩散；指数稳定；时间尺度

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双向联想记忆(BAM) 神经网络[1]已被成功应用于模拟识别与自动控制中. 与此同时，动态方程在时间尺度的研究已经引起了众多学者的关注[2-3]. 时间尺度理论的两个主要特征是统一与延伸，它允许同步处理微分方程和差分方程，这些理论的延伸就是所谓的动力学方程. 文献[4]介绍了在时间尺度上对动力学方程的研究过程和一般思路，所谓的时间尺度 T 就是实数集 $\mathbb{R}$ 的任意非空闭子集.

在时间尺度上研究带有狄利克雷边值条件和反应扩散项的 BAM 神经网络的稳定性具有十分重要的理论和应用意义，我们将会在时间尺度上研究以下带有反应扩散项和狄利克雷边值的 BAM 神经网络模型的稳定性.

$$\begin{cases} u_i^{\Delta}(t,\boldsymbol{x}) = \sum\limits_{k=1}^{l} \dfrac{\partial}{\partial x_k}\left(D_{ik}\dfrac{\partial u_i(t,\boldsymbol{x})}{\partial x_k}\right) - c_i u_i(t,\boldsymbol{x}) + \\ \qquad \sum\limits_{j=1}^{n} \omega_{ji} g_j(\nu_j(t,\boldsymbol{x})) + \sum\limits_{j=1}^{n} \omega_{ji}^{*} g_j(\nu_j(t-\tau_{ji},\boldsymbol{x})) + I_i \\ \nu_j^{\Delta}(t,\boldsymbol{x}) = \sum\limits_{k=1}^{l} \dfrac{\partial}{\partial x_k}\left(D_{jk}^{*}\dfrac{\partial \nu_j(t,\boldsymbol{x})}{\partial x_k}\right) - d_j \nu_j(t,\boldsymbol{x}) + \\ \qquad \sum\limits_{i=1}^{m} h_{ij} f_i(u_i(t,\boldsymbol{x})) + \sum\limits_{j=1}^{n} h_{ij}^{*} f_i(u_i(t-\gamma_{ij},\boldsymbol{x})) + J_j \end{cases} \tag{1}$$

其中：$i \in \{1,2,\cdots,m\}$，$j \in \{1,2,\cdots,n\}$，$t \in (0,+\infty)_T$，$\boldsymbol{x} = (x_1, x_2, \cdots, x_l)^{\mathrm{T}} \in \Omega \subset \mathbb{R}^l$，$\Omega$ 是一个带有光滑边界 $\partial\Omega$ 的有界紧集，在空间 $\mathbb{R}^l$ 中测度 $\Omega > 0$；$\boldsymbol{u} = (u_1, u_2, \cdots, u_m)^{\mathrm{T}} \in \mathbb{R}^m$，$\boldsymbol{v} = (\nu_1, \nu_2, \cdots, \nu_n)^{\mathrm{T}} \in \mathbb{R}^n$. $u_i(t,\boldsymbol{x})$ 和 $\nu_j(t,\boldsymbol{x})$ 分别表示第 i 个神经元和第 j 个神经元在 t 时刻和 $\boldsymbol{x}$ 空间中的状态；f_i 和 g_j 分别表示第 i 个神经元和第 j 个神经元在 t 时刻和 $\boldsymbol{x}$ 空间中的信号函数；I_i 和 J_j 分别表示第 i 个神经元和第 j 个神经元的外部输入函数；$c_i, d_j, \omega_{ji}, \omega_{ji}^{*}, h_{ij}, h_{ij}^{*}, \tau_{ji}, \gamma_{ij}$ 都是常数，$c_i > 0$ 和 $d_j > 0$ 分别表示第 i 个神经元和第 j 个神经元在进入封闭休息状态时重置自己潜力的速率，$\omega_{ji}, \omega_{ji}^{*}, h_{ij}$ 和 h_{ij}^{*} 表示它们之间的

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链接能力；光滑函数 $D_{ik}=D_{ik}(t,\boldsymbol{x},\boldsymbol{u})\geqslant 0$ 和 $D_{jk}^{*}=D_{jk}^{*}(t,\boldsymbol{x},\boldsymbol{u})\geqslant 0$ 分别表示第 i 个神经元和第 j 个神经元的传输扩散函数.

给出如下的边界条件和初始条件：

$$\begin{cases}u_i(t,\boldsymbol{x})=0 & (t,\boldsymbol{x})\in(0,+\infty)_T\times\partial\Omega \quad i=1,2,\cdots,m\\ \nu_j(t,\boldsymbol{x})=0 & (t,\boldsymbol{x})\in(0,+\infty)_T\times\partial\Omega \quad j=1,2,\cdots,n\end{cases}\tag{2}$$

$$\begin{cases}u_i(s,\boldsymbol{x})=\varphi_{ui}(s,\boldsymbol{x}) & (s,\boldsymbol{x})\in(0,+\infty)_T\times\Omega \quad i=1,2,\cdots,m\\ \nu_j(s,\boldsymbol{x})=\varphi_{vj}(s,\boldsymbol{x}) & (s,\boldsymbol{x})\in(0,+\infty)_T\times\Omega \quad j=1,2,\cdots,n\end{cases}\tag{3}$$

1 预备知识

先给出在后面证明过程中所需要的一些基本定义、引理.

引理 1[4] 假设函数 $f(t),g(t):T\longrightarrow\mathbb{R}$ 在 $t\in T^k$ 上是 Δ-可导的，则

$$(fg)^{\Delta}(t)=f^{\Delta}(t)g(t)+f(\sigma(t))g^{\Delta}(t)=f(t)g^{\Delta}(t)+f^{\Delta}(t)g(\sigma(t))$$

定义 1[5] 如果这里存在一个正常数 λ 和 $M=M(\lambda)\geqslant 1$，对于带有初始值 $(\boldsymbol{\varphi}^{u}(s,\boldsymbol{x}),\boldsymbol{\varphi}^{v}(s,\boldsymbol{x}))=(\varphi_1^u(s,\boldsymbol{x}),\varphi_2^u(s,\boldsymbol{x}),\cdots,\varphi_m^u(s,\boldsymbol{x}),\varphi_1^v(s,\boldsymbol{x}),\varphi_2^v(s,\boldsymbol{x}),\cdots,\varphi_n^v(s,\boldsymbol{x}))^{\mathrm{T}}\in C((-\infty,0]_T,\mathbb{R}^{m+n})$ 的系统(1)的任意解

$$(\boldsymbol{u}(t,\boldsymbol{x}),\boldsymbol{v}(t,\boldsymbol{x}))=(u_1(t,\boldsymbol{x}),u_2(t,\boldsymbol{x}),\cdots,u_m(t,\boldsymbol{x}),\nu_1(t,\boldsymbol{x}),\nu_2(t,\boldsymbol{x}),\cdots,\nu_n(t,\boldsymbol{x}))^{\mathrm{T}}$$

满足

$$\sqrt{\sum_{i=1}^{m}\|u_i(t,\boldsymbol{x})-u_i^{*}\|_2+\sum_{j=1}^{n}\|\nu_j(t,\boldsymbol{x})-v_j^{*}\|_2}\leqslant Me_{\Theta\lambda}(t,0),\ t\in T^{+}$$

则系统(1)的平衡点 $(\boldsymbol{u}^{*},\boldsymbol{v}^{*})=(u_1^{*},u_2^{*},\cdots,u_m^{*},\nu_1^{*},\nu_2^{*},\cdots,\nu_n^{*})^{\mathrm{T}}$ 被称之为全局指数稳定的.

定义 2[5] 对于每一个 $t\in T$，N 是 t 的一个邻域，则对于 $V\in C_{rd}(T\times\mathbb{R}^n,\mathbb{R}^{+})$，我们把 $D^{+}V^{\Delta}(t,\boldsymbol{x}(t))$ 定义为：给定 $\varepsilon>0$，存在 t 的一个右邻域 $N_{\varepsilon}\cap N$，对于每一 $s\in N_{\varepsilon}$，$s>t$，满足

$$\frac{1}{\mu(t,s)}(V(\sigma(t),\boldsymbol{x}(\sigma(t)))-V(s,\boldsymbol{x}(\sigma(t)))-\mu(s,t)f(t,\boldsymbol{x}(t)))<D^{+}V^{\Delta}(t,\boldsymbol{x}(t))+\varepsilon$$

这里

$$\mu(t,s)\equiv\sigma(t)-s$$

如果 t 是右离散的，并且 $V(t,\boldsymbol{x}(t))$ 关于 t 是连续的，这个就能够导出

$$D^{+}V^{\Delta}(t,\boldsymbol{x}(t))=\frac{V(\sigma(t),\boldsymbol{x}(\sigma(t)))-V(t,\boldsymbol{x}(\sigma(t)))}{\sigma(t)-t}$$

引理 2[6] Ω 是 $|x_i|\leqslant l_i\ (i=1,2,\cdots,m)$ 的一个立方体，$h(\boldsymbol{x})$ 是一个属于 $C^1(\Omega)$ 的实值函数并且消失在 Ω 的边界 $\partial\Omega$ 上，等价于 $h(\boldsymbol{x})|_{\partial\Omega}=0$，则

$$\int_{\Omega}h^2(\boldsymbol{x})\mathrm{d}\boldsymbol{x}=l_i^2\int_{\Omega}\left|\frac{\partial h}{\partial x_i}\right|^2\mathrm{d}\boldsymbol{x}$$

为了方便应用，我们介绍一些概念：把 $\boldsymbol{x}\in\mathbb{R}^{m+n}$ 的向量范数定义为

$$\|\boldsymbol{x}\|_1=\sum_{i=1}^{m+n}|x_i|,\ C_{rd}=C_{rd}((-\infty,0]_T\times\mathbb{R}^l,\mathbb{R}^{m+n})$$

是由 $(-\infty,0]_T\times\mathbb{R}^l$ 映射到 $\mathbb{R}^{m+n}$ 上的 rd-连续函数组成的具有一致收敛拓扑的 Banach 空间. Ω 是 $\mathbb{R}^l$ 中的一个带有光滑边界的有界开区域，测度 $\Omega>0$. $L^2(\Omega)$ 是由 Ω 上的实函数组成的 Banach 空间，L^2 是 Lebesgue 可测的，并且它由以下的范数构成一个 Banach 空间，

$$\|(\boldsymbol{u}(t),\boldsymbol{v}(t))\|_2=\sqrt{\sum_{i=1}^{m}\|u_i(t)\|_2^2+\sum_{j=1}^{n}\|\nu_j(t)\|_2^2}$$

这里

$$\boldsymbol{u}(t)=(u_1(t),u_2(t),\cdots,u_m(t))^{\mathrm{T}}\qquad \boldsymbol{v}(t)=(\nu_1(t),\nu_2(t),\cdots,\nu_n(t))^{\mathrm{T}}$$

$$\|u_i(t)\|_2=\left(\int_{\Omega}u_i^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x}\right)^{\frac{1}{2}}\qquad \|\nu_j(t)\|_2=\left(\int_{\Omega}\nu_j^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x}\right)^{\frac{1}{2}}$$

对任意的 $(\boldsymbol{\varphi}^u(s,\boldsymbol{x}),\boldsymbol{\varphi}^v(s,\boldsymbol{x}))\in((-\infty,0]_T\times\Omega,\mathbb{R}^{m+n})$，我们定义

$$\|(\boldsymbol{\varphi}^u,\boldsymbol{\varphi}^v)\|_2=\sqrt{\sum_{i=1}^m\|\varphi_i^u\|_2^2+\sum_{j=1}^n\|\varphi_j^v\|_2^2}$$

这里

$$\boldsymbol{\varphi}^u(s,\boldsymbol{x})=(\varphi_1^u(s,\boldsymbol{x}),\varphi_2^u(s,\boldsymbol{x}),\cdots,\varphi_m^u(s,\boldsymbol{x}))^{\mathrm{T}}$$

$$\boldsymbol{\varphi}^v(s,\boldsymbol{x})=(\varphi_1^v(s,\boldsymbol{x}),\varphi_2^v(s,\boldsymbol{x}),\cdots,\varphi_n^v(s,\boldsymbol{x}))^{\mathrm{T}}$$

$$\|\varphi_i^u\|_2=\left(\int_\Omega|\varphi_i^u(\cdot,\boldsymbol{x})|_\infty^2\mathrm{d}\boldsymbol{x}\right)^{\frac{1}{2}}\qquad\|\varphi_i^v\|_2=\left(\int_\Omega|\varphi_i^v(\cdot,\boldsymbol{x})|_\infty^2\mathrm{d}\boldsymbol{x}\right)^{\frac{1}{2}}$$

$$|\varphi_i^u(\cdot,\boldsymbol{x})|_\infty=\sup_{s\in(-\infty,0]}|\varphi_i^u(s,\boldsymbol{x})|\qquad|\varphi_i^v(\cdot,\boldsymbol{x})|_\infty=\sup_{s\in(-\infty,0]}|\varphi_i^v(s,\boldsymbol{x})|$$

$$\tau=\max\{\tau_{ji},\gamma_{ij}\}$$

2　主要结论

为了在时间尺度上研究带有反应扩散项和狄利克雷边值的 BAM 神经网络模型的稳定性，我们做如下的假设：

(H_1)：神经功能函数 $f_i(\cdot)$ 和 $g_j(\cdot)$ 都是 Lipschitz 连续的，即这里存在常数 F_i 和 G_j，对任意的 $u_1,u_2\in\mathbb{R}$ 使其满足 $|f_i(u_1)-f_i(u_2)|\leqslant F_i|u_1-u_2|$，$|g_j(u_1)-g_j(u_2)|\leqslant G_j|u_1-u_2|$.

(H_2) $c_i>F_i\sum_{j=1}^n(|h_{ij}|+|h_{ij}^*|)$，$i=1,2,\cdots,m$.

(H_3) $d_j>G_j\sum_{i=1}^m(|\omega_{ji}|+|\omega_{ji}^*|)$，$j=1,2,\cdots,n$.

(H_4) $-2\sum_{k=1}^l\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^n|\omega_{ji}|G_j+\sum_{j=1}^n|h_{ij}|F_i+2\sum_{j=1}^n|\omega_{ji}^*|G_j+2\sum_{j=1}^n|h_{ij}^*|F_ie_{1\oplus1}(\gamma_{ij},0)<0$.

(H_5) $-2\sum_{k=1}^l\frac{D_{jk}^*}{l_k^2}-2d_j+\sum_{i=1}^m|h_{ij}|F_i+\sum_{i=1}^m|\omega_{ji}|G_j+2\sum_{i=1}^m|h_{ij}^*|F_i+2\sum_{i=1}^m|\omega_{ji}^*|G_je_{1\oplus1}(\tau_{ji},0)<0$.

定理 1　如果条件 $(\mathrm{H}_1)-(\mathrm{H}_5)$ 都成立，则系统(1) 的平衡点 $(\boldsymbol{u}^*,\boldsymbol{v}^*)=(u_1^*,u_2^*,\cdots,u_m^*,v_1^*,v_2^*,\cdots,v_n^*)^{\mathrm{T}}$ 是全局指数稳定的.

证　假设

$$(\boldsymbol{u}(t,\boldsymbol{x}),\boldsymbol{v}(t,\boldsymbol{x}))=(u_1(t,\boldsymbol{x}),u_2(t,\boldsymbol{x}),\cdots,u_m(t,\boldsymbol{x}),v_1(t,\boldsymbol{x}),v_2(t,\boldsymbol{x}),\cdots,v_n(t,\boldsymbol{x}))^{\mathrm{T}}$$

是系统(1) 带有初值条件 $(\boldsymbol{\varphi}^u(s,\boldsymbol{x}),\boldsymbol{\varphi}^v(s,\boldsymbol{x}))\in C_{rd}((-\infty,0]_T\times\Omega,\mathbb{R}^{m+n})$ 的任意解，$(\boldsymbol{u}^*,\boldsymbol{v}^*)=(u_1^*,u_2^*,\cdots,u_m^*,v_1^*,v_2^*,\cdots,v_n^*)$ 是系统(1) 的平衡点，定义

$$(\boldsymbol{z}(t,\boldsymbol{x}),\boldsymbol{y}(t,\boldsymbol{x}))=(\boldsymbol{u}(t,\boldsymbol{x})-\boldsymbol{u}^*,\boldsymbol{v}(t,\boldsymbol{x})-\boldsymbol{v}^*)$$

$$(\boldsymbol{\varphi}^z(s,\boldsymbol{x}),\boldsymbol{\varphi}^y(s,\boldsymbol{x}))=(\boldsymbol{\varphi}^u(s,\boldsymbol{x})-\boldsymbol{u}^*,\boldsymbol{\varphi}^v(s,\boldsymbol{x})-\boldsymbol{v}^*)$$

重新改写系统(1) 为以下形式：

$$\begin{cases}z_i^\Delta(t,\boldsymbol{x})=\sum_{k=1}^l\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iz_i(t,\boldsymbol{x})+\sum_{j=1}^n\omega_{ji}(g_j(y_j(t,\boldsymbol{x})+v_j^*)-g_j(v_j^*))+\\\qquad\sum_{j=1}^n\omega_{ji}^*(g_j(y_j(t-\tau_{ji},\boldsymbol{x})+v_j^*)-g_j(v_j^*))\\y_j^\Delta(t,\boldsymbol{x})=\sum_{k=1}^l\frac{\partial}{\partial x_k}\left(D_{jk}^*\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\right)-d_jy_j(t,\boldsymbol{x})+\sum_{i=1}^m h_{ij}(f_i(z_i(t,\boldsymbol{x})+u_i^*)-f_i(u_i^*))+\\\qquad\sum_{i=1}^m h_{ij}^*(f_i(z_i(t-\gamma_{ij},\boldsymbol{x})+u_i^*)-f_i(u_i^*))\end{cases}\tag{4}$$

依照系统(4) 计算 $\|z_i(t,\cdot)\|_2^2$ 和 $\|y_j(t,\cdot)\|_2^2$ 的 Δ-微分，我们得到

$$(\|z_i(t,\cdot)\|_2^2)^\Delta=\int_\Omega(|z_i(t,\boldsymbol{x})|^2)^\Delta\mathrm{d}\boldsymbol{x}=$$

$$2\sum_{k=1}^{l}\int_{\Omega}z_i(t,\boldsymbol{x})\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\right)\mathrm{d}\boldsymbol{x}+$$
$$2\sum_{j=1}^{n}\omega_{ji}\int_{\Omega}z_i(t,\boldsymbol{x})(g_j(y_j(t,\boldsymbol{x})+\nu_j^*)-g_j(\nu_j^*))\,\mathrm{d}\boldsymbol{x}+$$
$$2\sum_{j=1}^{n}\omega_{ji}^*\int_{\Omega}z_i(t,\boldsymbol{x})(g_j(y_j(t-\tau_{ji},\boldsymbol{x})+\nu_j^*)-g_j(\nu_j^*))\,\mathrm{d}\boldsymbol{x}+\mu(t)\|z_i^{\Delta}(t,\cdot)\|_2^2-$$
$$2c_i\int_{\Omega}z_i^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x}\qquad i=1,2,\cdots,m \tag{5}$$

并且

$$(\|y_j(t,\cdot)\|_2^2)^{\Delta}=\int_{\Omega}(|y_j(t,\boldsymbol{x})|^2)^{\Delta}\mathrm{d}\boldsymbol{x}=$$
$$2\sum_{k=1}^{l}\int_{\Omega}y_j(t,\boldsymbol{x})\frac{\partial}{\partial x_k}\left(D_{jk}^*\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\right)\mathrm{d}\boldsymbol{x}+$$
$$2\sum_{i=1}^{m}h_{ij}\int_{\Omega}y_j(t,\boldsymbol{x})(f_i(z_i(t,\boldsymbol{x})+u_i^*)-f_i(u_i^*))\,\mathrm{d}\boldsymbol{x}+$$
$$2\sum_{i=1}^{m}h_{ij}^*\int_{\Omega}y_j(t,\boldsymbol{x})(f_i(z_i(t-\gamma_{ij},\boldsymbol{x})+u_i^*)-f_i(u_i^*))\,\mathrm{d}\boldsymbol{x}+\mu(t)\|y_j^{\Delta}(t,\cdot)\|_2^2-$$
$$2d_j\int_{\Omega}y_j^2(t,\boldsymbol{x})\mathrm{d}\boldsymbol{x}\qquad j=1,2,\cdots,n \tag{6}$$

由格林公式、狄利克雷边值条件和引理(2)，我们得到：

$$\sum_{k=1}^{l}\int_{\Omega}z_i(t,\boldsymbol{x})\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\right)\mathrm{d}\boldsymbol{x}=$$
$$\sum_{k=1}^{l}\int_{\partial\Omega}z_i(t,\boldsymbol{x})D_{ik}\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\mathrm{dS}-\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}=$$
$$-\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial z_i(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}\leqslant-\sum_{k=1}^{l}\int_{\Omega}\frac{D_{ik}}{l_k^2}(z_i(t,\boldsymbol{x}))^2\mathrm{d}\boldsymbol{x} \tag{7}$$

$$\sum_{k=1}^{l}\int_{\Omega}y_j(t,\boldsymbol{x})\frac{\partial}{\partial x_k}\left(D_{jk}^*\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\right)\mathrm{d}\boldsymbol{x}=$$
$$\sum_{k=1}^{l}\int_{\partial\Omega}y_j(t,\boldsymbol{x})D_{jk}^*\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\mathrm{dS}-\sum_{k=1}^{l}\int_{\Omega}D_{jk}^*\left(\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}=$$
$$-\sum_{k=1}^{l}\int_{\Omega}D_{jk}^*\left(\frac{\partial y_j(t,\boldsymbol{x})}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}\leqslant-\sum_{k=1}^{l}\int_{\Omega}\frac{D_{jk}^*}{l_k^2}(y_j(t,\boldsymbol{x}))^2\mathrm{d}\boldsymbol{x} \tag{8}$$

由条件(H_1)，(H_2)，Hölder 不等式和等式(4)－(8) 可得：

$$(\|z_i(t,\cdot)\|_2^2)^{\Delta}\leqslant-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}\|z_i(t,\cdot)\|_2^2-2c_i\|z_i(t,\cdot)\|_2^2+$$
$$\sum_{j=1}^{n}|\omega_{ji}|G_j\|z_i(t,\cdot)\|_2^2+\sum_{j=1}^{n}|\omega_{ji}|G_j\|y_j(t,\cdot)\|_2^2+$$
$$2\sum_{j=1}^{n}|\omega_{ji}^*|G_j\int_{\Omega}|z_i(t,\boldsymbol{x})||y_j(t-\tau_{ji},\boldsymbol{x})|\mathrm{d}\boldsymbol{x}+\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2 \tag{9}$$

其中

$$\|(z_i(t,\cdot))^{\Delta}\|_2^2=q_i(t)\|z_i(t,\cdot)\|_2^2\qquad q_i(t)\geqslant 0\qquad i=1,2,\cdots,m$$
$$(\|y_j(t,\cdot)\|_2^2)^{\Delta}\leqslant-2\sum_{k=1}^{l}\frac{D_{jk}^*}{l_k^2}\|y_j(t,\cdot)\|_2^2-2d_j\|y_j(t,\cdot)\|_2^2+$$
$$\sum_{i=1}^{m}F_i|h_{ij}|\|y_j(t,\cdot)\|_2^2+\sum_{i=1}^{m}F_i|h_{ij}|\|z_i(t,\cdot)\|_2^2+$$

$$2\sum_{i=1}^{m}|h_{ij}^{*}|G_j\int_{\Omega}F_i|y_j(t,\boldsymbol{x})||z_i(t-\gamma_{ij},\boldsymbol{x})|\mathrm{d}\boldsymbol{x}+\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2 \tag{10}$$

其中

$$\|(y_j(t,\cdot))^{\Delta}\|_2^2=p_j(t)\|y_j(t,\cdot)\|_2^2 \qquad p_j(t)\geqslant 0 \qquad j=1,2,\cdots,n$$

如果条件(H_4)和(H_5)都满足，我们就能够选择一个正数 $\sigma_{i1}>0$ 和 $\sigma_{j2}>0$(可能非常小)，$i=1,2,\cdots,m$，$j=1,2,\cdots,n$. 使其满足以下方程.

$$-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\sum_{j=1}^{n}|h_{ij}|F_i+2\sum_{j=1}^{n}|\omega_{ji}^{*}|G_j+2\sum_{j=1}^{n}|h_{ij}^{*}|F_ie_{1\oplus 1}(\gamma_{ij},0)+\sigma_{i1}<0 \tag{11}$$

和

$$-2\sum_{k=1}^{l}\frac{D_{jk}^{*}}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+\sum_{i=1}^{m}|\omega_{ji}|G_j+2\sum_{i=1}^{m}|h_{ij}^{*}|F_i+2\sum_{i=1}^{m}|\omega_{ji}^{*}|G_je_{1\oplus 1}(\tau_{ji},0)+\sigma_{j2}<0 \tag{12}$$

接下来考虑以下函数：

$$\begin{aligned}\widetilde{p}_{i1}(x_i)=&(x_i\oplus x_i)-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\\&\sum_{j=1}^{n}|h_{ij}|F_i+2\sum_{j=1}^{n}|\omega_{ji}^{*}|G_j+2\sum_{j=1}^{n}|h_{ij}^{*}|F_ie_{1\oplus 1}(\gamma_{ij},0)+\\&\omega(x_i)\mu(t)q_i(t)\frac{\max\{e_{(\omega(x_i)-1)\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2}(t,0),e_{x_i\oplus x_i}(\sigma(t),0)\}}{e_{x_i\oplus x_i}(\sigma(t),0)}\end{aligned} \tag{13}$$

$$\begin{aligned}\widetilde{p}_{j2}(x_j)=&(x_j\oplus x_j)-2\sum_{k=1}^{l}\frac{D_{jk}^{*}}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+\\&\sum_{i=1}^{m}|\omega_{ji}|G_j+2\sum_{i=1}^{m}|h_{ij}^{*}|F_i+2\sum_{i=1}^{m}|\omega_{ji}^{*}|G_je_{1\oplus 1}(\tau_{ij},0)+\\&\omega(x_j)\mu(t)p_j(t)\frac{\max\{e_{(\omega(x_j)-1)\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2}(t,0),e_{x_j\oplus x_j}(\sigma(t),0)\}}{e_{x_j\oplus x_j}(\sigma(t),0)}\end{aligned} \tag{14}$$

这里 $\omega(\boldsymbol{x})=\int_0^x\frac{\mathrm{e}^{\boldsymbol{x}-s}}{\boldsymbol{x}-s}\mathrm{d}s$，$\boldsymbol{x}\in[0,+\infty)$，$i=1,2,\cdots,m$，$j=1,2,\cdots,n$. 由等式(11)和(13)可知 $\widetilde{p}_{i1}(0)<-\sigma_{i1}<0$，而且 $\widetilde{p}_{i1}(x_i)$ 对于每个 $x_i\in[0,+\infty)$ 是连续的，当 $x_i\to+\infty$ 时 $\widetilde{p}_{i1}(x_i)\to+\infty$，因此这里存在一个正常数 $\varepsilon_i\in(0,+\infty)$ 满足

$$\begin{aligned}\widetilde{p}_{i1}(\varepsilon_i)=&(\varepsilon_i\oplus\varepsilon_i)-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\\&\sum_{j=1}^{n}|h_{ij}|F_i+2\sum_{j=1}^{n}|\omega_{ji}^{*}|G_j+2\sum_{j=1}^{n}|h_{ij}^{*}|F_ie_{1\oplus 1}(\gamma_{ij},0)+\\&\omega(\varepsilon_i)\mu(t)q_i(t)\frac{\max\{e_{(\omega(\varepsilon_i)-1)\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2}(t,0),e_{\varepsilon_i\oplus\varepsilon_i}(\sigma(t),0)\}}{e_{\varepsilon_i\oplus\varepsilon_i}(\sigma(t),0)}\leqslant 0\end{aligned} \tag{15}$$

$$\begin{aligned}\widetilde{p}_{i1}(\varepsilon_i^{*})=&(\varepsilon_i^{*}\oplus\varepsilon_i^{*})-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\sum_{j=1}^{n}|h_{ij}|F_i+\\&2\sum_{j=1}^{n}|\omega_{ji}^{*}|G_j+2\sum_{j=1}^{n}|h_{ij}^{*}|F_ie_{1\oplus 1}(\gamma_{ij},0)+\\&\omega(\varepsilon_i^{*})\mu(t)q_i(t)\frac{\max\{e_{(\omega(\varepsilon_i^{*})-1)\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2}(t,0),e_{\varepsilon_i^{*}\oplus\varepsilon_i^{*}}(\sigma(t),0)\}}{e_{\varepsilon_i^{*}\oplus\varepsilon_i^{*}}(\sigma(t),0)}=0\end{aligned} \tag{16}$$

这里 $\varepsilon_i\in(0,\varepsilon_i^{*})\cap(0,1)$，$i=1,2,\cdots,m$. 由等式(12)和(14)，我们也能够得知 $\widetilde{p}_{j2}(0)<-\sigma_{j2}<0$ 且 $\widetilde{p}_{j2}(x_j)$ 对于每个 $x_j\in[0,+\infty)$ 是连续的，当 $x_j\to+\infty$ 时，$\widetilde{p}_{j2}(x_j)\to+\infty$，因此这里存在一个正常数 $\eta_j\in(0,+\infty)$ 满足：

$$\widetilde{p}_{j2}(\eta_j)=(\eta_j\oplus\eta_j)-2\sum_{k=1}^{l}\frac{D_{jk}^*}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+$$
$$\sum_{i=1}^{m}|\omega_{ji}|G_j+2\sum_{i=1}^{m}|h_{ij}^*|F_i+2\sum_{i=1}^{m}|\omega_{ji}^*|G_je_{1\oplus1}(\tau_{ij},0)+$$
$$\omega(\eta_j)\mu(t)p_j(t)\frac{\max\{e_{(\omega(\eta_j)-1)\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2}(t,0),e_{\eta_j\oplus\eta_j}(\sigma(t),0)\}}{e_{\eta_j\oplus\eta_j}(\sigma(t),0)}\leqslant0 \tag{17}$$

$$\widetilde{p}_{j2}(\eta_j^*)=(\eta_j^*\oplus\eta_j^*)-2\sum_{k=1}^{l}\frac{D_{jk}^*}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+\sum_{i=1}^{m}|\omega_{ji}|G_j+$$
$$2\sum_{i=1}^{m}|h_{ij}^*|F_i+2\sum_{i=1}^{m}|\omega_{ji}^*|G_je_{1\oplus1}(\tau_{ij},0)+$$
$$\omega(\eta_j^*)\mu(t)p_j(t)\frac{\max\{e_{(\omega(\eta_j^*)-1)\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2}(t,0),e_{\eta_j^*\oplus\eta_j^*}(\sigma(t),0)\}}{e_{\eta_j^*\oplus\eta_j^*}(\sigma(t),0)}=0 \tag{18}$$

这里 $\eta_j\in(0,\eta_j^*)\cap(0,1)$，$j=1,2,\cdots,n$. 令 $\varepsilon=\min\limits_{1\leqslant i\leqslant m,\,1\leqslant j\leqslant n}\{\varepsilon_i,\eta_j\}$，显然 $0<\varepsilon<1$，我们得到

$$\widetilde{p}_{i1}(\varepsilon)=(\varepsilon\oplus\varepsilon)-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\sum_{j=1}^{n}|h_{ij}|F_i+2\sum_{j=1}^{n}|\omega_{ji}^*|G_j+2\sum_{j=1}^{n}|h_{ij}^*|F_ie_{1\oplus1}(\gamma_{ij},0)+$$
$$\omega(\varepsilon)\mu(t)q_i(t)\frac{\max\{e_{(\omega(\varepsilon)-1)\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2}(t,0),e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\}}{e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)}\leqslant0 \tag{19}$$

$$\widetilde{p}_{j2}(\varepsilon)=(\varepsilon\oplus\varepsilon)-2\sum_{k=1}^{l}\frac{D_{jk}^*}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+\sum_{i=1}^{m}|\omega_{ji}|G_j+2\sum_{i=1}^{m}|h_{ij}^*|F_i+2\sum_{i=1}^{m}|\omega_{ji}^*|G_je_{1\oplus1}(\tau_{ij},0)+$$
$$\omega(\varepsilon)\mu(t)p_j(t)\frac{\max\{e_{(\omega(\varepsilon)-1)\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2}(t,0),e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\}}{e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)}\leqslant0 \tag{20}$$

现在我们构造一个李雅普诺夫函数：

$$V(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=V_1(t,\boldsymbol{z}(t),\boldsymbol{y}(t))+V_2(t,\boldsymbol{z}(t),\boldsymbol{y}(t))+V_3(t,\boldsymbol{z}(t),\boldsymbol{y}(t)) \tag{21}$$
$$V_1(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=\sum_{i=1}^{m}e_{\varepsilon\oplus\varepsilon}(t,0)\|z_i(t,\cdot)\|_2^2+\sum_{j=1}^{n}e_{\varepsilon\oplus\varepsilon}(t,0)\|y_j(t,\cdot)\|_2^2$$
$$V_2(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=\sum_{i=1}^{m}e_{(\omega(\varepsilon)-1)\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2}(t,0)+\sum_{j=1}^{n}e_{(\omega(\varepsilon)-1)\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2}(t,0)$$
$$V_3(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=2\sum_{i=1}^{m}\sum_{j=1}^{n}|\omega_{ji}^*|G_j\int_\Omega\int_{t-\tau_{ji}}^{t}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\tau_{ji}),0)|y_j(\tau,\boldsymbol{x})|^2\Delta\tau\mathrm{d}\boldsymbol{x}+$$
$$2\sum_{i=1}^{m}\sum_{j=1}^{n}|h_{ij}^*|F_i\int_\Omega\int_{t-\gamma_{ij}}^{t}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\gamma_{ij}),0)|z_i(\tau,\boldsymbol{x})|^2\Delta\tau\mathrm{d}\boldsymbol{x}$$

依照系统(4)，计算 $D^+V^\Delta(t)$[5]，得到

$$D^+V_1^\Delta(t,\boldsymbol{z}(t),\boldsymbol{y}(t))\leqslant$$
$$(\varepsilon\oplus\varepsilon)e_{\varepsilon\oplus\varepsilon}(t,0)\Big\{\sum_{i=1}^{m}\|z_i(t,\cdot)\|_2^2+\sum_{j=1}^{n}\|y_j(t,\cdot)\|_2^2\Big\}+$$
$$e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{i=1}^{m}\Big\{-2\sum_{k=1}^{l}\frac{D_{ik}}{l_k^2}-2c_i+\sum_{j=1}^{n}|\omega_{ji}|G_j+\sum_{j=1}^{n}|h_{ij}|F_i\Big\}\|z_i(t,\cdot)\|_2^2+$$
$$e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{j=1}^{n}\Big\{-2\sum_{k=1}^{l}\frac{D_{jk}^*}{l_k^2}-2d_j+\sum_{i=1}^{m}|h_{ij}|F_i+\sum_{i=1}^{m}|\omega_{ji}|G_j\Big\}\|y_j(t,\cdot)\|_2^2+$$
$$2e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{i=1}^{m}\sum_{j=1}^{n}|\omega_{ji}^*|G_j\int_\Omega|z_i(t,\boldsymbol{x})||y_j(t-\tau_{ji},\boldsymbol{x})|\mathrm{d}\boldsymbol{x}+$$
$$2e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{i=1}^{m}\sum_{j=1}^{n}|h_{ij}^*|F_i\int_\Omega|y_j(t,\boldsymbol{x})||z_i(t-\gamma_{ij},\boldsymbol{x})|\mathrm{d}\boldsymbol{x}+$$
$$e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{i=1}^{m}\mu(t)q_i(t)\|z_i(t,\cdot)\|_2^2+e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{j=1}^{n}\mu(t)p_j(t)\|y_j(t,\cdot)\|_2^2 \tag{22}$$

和

$$D^{+}V_{2}^{\Delta}(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=\sum_{i=1}^{m}(\omega(\varepsilon)-1)\mu(t)q_{i}(t)\|z_{i}(t,\cdot)\|_{2}^{2}e_{(\omega(\varepsilon)-1)\mu(t)q_{i}(t)\|z_{i}(t,\cdot)\|_{2}^{2}}(t,0)+$$

$$\sum_{j=1}^{n}(\omega(\varepsilon)-1)\mu(t)p_{j}(t)\|y_{j}(t,\cdot)\|_{2}^{2}e_{(\omega(\varepsilon)-1)\mu(t)p_{j}(t)\|y_{j}(t,\cdot)\|_{2}^{2}}(t,0)\tag{23}$$

$$\begin{aligned}D^{+}V_{3}^{\Delta}(t,\boldsymbol{z}(t),\boldsymbol{y}(t))=&2\sum_{i=1}^{m}\sum_{j=1}^{n}G_{j}|\omega_{ji}^{*}|\int_{\Omega}e_{\varepsilon\oplus\varepsilon}(\sigma(t+\tau_{ji}),0)|y_{j}(t,\boldsymbol{x})|^{2}\mathrm{d}\boldsymbol{x}-\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}G_{j}|\omega_{ji}^{*}|\int_{\Omega}e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)|y_{j}(t-\tau_{ji},\boldsymbol{x})|^{2}\mathrm{d}\boldsymbol{x}+\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}F_{i}|h_{ij}^{*}|\int_{\Omega}e_{\varepsilon\oplus\varepsilon}(\sigma(t+\gamma_{ij}),0)|z_{i}(t,\boldsymbol{x})|^{2}\mathrm{d}\boldsymbol{x}-\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}F_{i}|h_{ij}^{*}|\int_{\Omega}e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)|z_{i}(t-\gamma_{ij},\boldsymbol{x})|^{2}\mathrm{d}\boldsymbol{x}\end{aligned}\tag{24}$$

通过等式(22)－(24)、条件(H_4)和(H_5)，推导出

$$\begin{aligned}&D^{+}V^{\Delta}(t,\boldsymbol{z}(t),\boldsymbol{y}(t))\leqslant\\&e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{i=1}^{m}\Bigg\{(\varepsilon\oplus\varepsilon)+\omega(\varepsilon)\mu(t)q_{i}(t)\frac{\max\{e_{(\omega(\varepsilon)-1)\mu(t)q_{i}(t)\|z_{i}(t,0)\|_{2}^{2}}(t,0),e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\}}{e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)}-\\&2\sum_{k=1}^{l}\frac{D_{ik}}{l_{k}^{2}}-2c_{i}+\sum_{j=1}^{n}|\omega_{ji}|G_{j}+\sum_{j=1}^{n}|h_{ij}|F_{i}+2\sum_{j=1}^{n}G_{j}|\omega_{ji}^{*}|+2\sum_{j=1}^{n}|h_{ij}^{*}|F_{i}e_{1\oplus1}(\gamma_{ij},0)\Bigg\}\|z_{i}(t,\cdot)\|_{2}^{2}+\\&e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\sum_{j=1}^{n}\Bigg\{(\varepsilon\oplus\varepsilon)+\omega(\varepsilon)\mu(t)p_{j}(t)\frac{\max\{e_{(\omega(\varepsilon)-1)\mu(t)p_{j}(t)\|y_{j}(t,0)\|_{2}^{2}}(t,0),e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)\}}{e_{\varepsilon\oplus\varepsilon}(\sigma(t),0)}-\\&2\sum_{k=1}^{l}\frac{D_{jk}^{*}}{l_{k}^{2}}-2d_{j}+\sum_{i=1}^{m}|h_{ij}|F_{i}+\sum_{i=1}^{m}|\omega_{ji}|G_{j}+\sum_{i=1}^{m}F_{i}|h_{ij}^{*}|+2\sum_{i=1}^{m}|\omega_{ji}^{*}|G_{j}e_{1\oplus1}(\tau_{ji},0)\Bigg\}\|y_{j}(t,\cdot)\|_{2}^{2}\leqslant0\end{aligned}\tag{25}$$

由不等式(22)－(25)可知，对于所有 $t\in[0,+\infty)_{T}$ 有：

$$\sum_{i=1}^{m}e_{\varepsilon\oplus\varepsilon}(t,0)\|z_{i}(t,\cdot)\|_{2}^{2}+\sum_{j=1}^{n}e_{\varepsilon\oplus\varepsilon}(t,0)\|y_{j}(t,\cdot)\|_{2}^{2}\leqslant V(t,\boldsymbol{z}(t),\boldsymbol{y}(t))\leqslant V(0,\boldsymbol{z}(0),\boldsymbol{y}(0))\tag{26}$$

$$\begin{aligned}V(0,\boldsymbol{z}(0),\boldsymbol{y}(0))\leqslant&\sum_{i=1}^{m}\|\varphi_{i\tau}^{z}\|_{2}^{2}+\sum_{j=1}^{n}\|\varphi_{j\tau}^{y}\|_{2}^{2}+m+n+\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}G_{j}|\omega_{ji}^{*}|\|\varphi_{j\tau}^{y}\|_{2}^{2}\int_{-\tau_{ji}}^{0}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\tau_{ji}),0)\Delta\tau+\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}F_{i}|h_{ij}^{*}|\|\varphi_{i\tau}^{z}\|_{2}^{2}\int_{-\gamma_{ij}}^{0}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\gamma_{ij}),0)\Delta\tau\end{aligned}\tag{27}$$

由不等式(26)和(27)可知：

$$\sqrt{\sum_{i=1}^{m}\|z_{i}(t,\cdot)\|_{2}^{2}+\sum_{j=1}^{n}\|y_{j}(t,\cdot)\|_{2}^{2}}\leqslant Me_{\Theta\varepsilon}(t,0)\tag{28}$$

这里

$$\begin{aligned}M=\mathrm{sqrt}\Bigg\{&\sum_{i=1}^{m}\|\varphi_{i\tau}^{z}\|_{2}^{2}+\sum_{j=1}^{n}\|\varphi_{j\tau}^{y}\|_{2}^{2}+m+n+2\sum_{i=1}^{m}\sum_{j=1}^{n}G_{j}|\omega_{ji}^{*}|\|\varphi_{j\tau}^{y}\|_{2}^{2}\int_{-\tau_{ji}}^{0}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\tau_{ji}),0)\Delta\tau+\\&2\sum_{i=1}^{m}\sum_{j=1}^{n}F_{i}|h_{ij}^{*}|\|\varphi_{i\tau}^{z}\|_{2}^{2}\int_{-\gamma_{ij}}^{0}e_{\varepsilon\oplus\varepsilon}(\sigma(\tau+\gamma_{ij}),0)\Delta\tau\Bigg\}\end{aligned}$$

因此，通过定义 4 可知，系统(1)的平衡点是全局指数稳定的. 证毕.

3　举　　例

例 1　考虑以下具有狄利克雷边值条件和反应扩散项的 BAM 神经网络模型平衡点的全局指数稳定性：

$$\begin{cases} u_i^{\Delta}(t,\boldsymbol{x})=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,\boldsymbol{x})}{\partial x_k}\right)-c_iu_i(t,\boldsymbol{x})+\sum_{j=1}^{n}\omega_{ji}g_j(\nu_j(t,\boldsymbol{x}))+ \\ \qquad \sum_{j=1}^{n}\omega_{ji}^{*}g_j(\nu_j(t-\tau_{ji},\boldsymbol{x}))+I_i,\ (t,\boldsymbol{x})\in[0,+\infty)_T\times\Omega \\ \nu_j^{\Delta}(t,\boldsymbol{x})=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{jk}^{*}\frac{\partial \nu_j(t,\boldsymbol{x})}{\partial x_k}\right)-d_j\nu_j(t,\boldsymbol{x})+\sum_{i=1}^{m}h_{ij}f_i(u_i(t,\boldsymbol{x}))+ \\ \qquad \sum_{j=1}^{n}h_{ij}^{*}f_i(u_i(t-\gamma_{ij},\boldsymbol{x}))+J_j,\ (t,\boldsymbol{x})\in[0,+\infty)_T\times\Omega \\ u_i(t,\boldsymbol{x})=0,\ (t,\boldsymbol{x})\in T\times\partial\Omega, \\ \nu_j(t,\boldsymbol{x})=0,\ (t,\boldsymbol{x})\in T\times\partial\Omega, \\ u_i(s,\boldsymbol{x})=\varphi_i^{u}(s,\boldsymbol{x}),\ (s,\boldsymbol{x})\in[0,+\infty)_T\times\Omega \\ \nu_j(s,\boldsymbol{x})=\varphi_j^{\nu}(s,\boldsymbol{x}),\ (s,\boldsymbol{x})\in[0,+\infty)_T\times\Omega \end{cases} \tag{29}$$

其中：$i,j=1,2$；这里 $\Omega=\{\boldsymbol{x}:|\boldsymbol{x}|<1,\ l=1,2.\}$.

$$(D_{ik})_{2\times2}=\begin{pmatrix}\frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{4}\end{pmatrix}\qquad (D_{jk}^{*})_{2\times2}=\begin{pmatrix}\frac{1}{4} & \frac{1}{2}\\ \frac{1}{2} & 1\end{pmatrix}\qquad (c_i)_{2\times1}=\begin{pmatrix}\frac{3}{4}\\ \frac{1}{2}\end{pmatrix}$$

$$(d_j)_{2\times1}=\begin{pmatrix}\frac{1}{2}\\ 1\end{pmatrix}\qquad (\omega_{ji})_{2\times2}=\begin{pmatrix}\frac{1}{4} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{4}\end{pmatrix}\qquad (h_{ij})_{2\times2}=\begin{pmatrix}\frac{1}{2} & \frac{1}{4}\\ \frac{1}{4} & \frac{1}{2}\end{pmatrix}$$

$$(\omega_{ji})_{2\times2}=\begin{pmatrix}\frac{1}{27} & \frac{1}{18}\\ \frac{1}{18} & \frac{1}{27}\end{pmatrix}\qquad (h_{ij}^{*})_{2\times2}=\begin{pmatrix}\frac{1}{16} & \frac{1}{16}\\ \frac{1}{16} & \frac{1}{16}\end{pmatrix}\qquad (K_{ji}(s))_{2\times2}=\begin{pmatrix}8\mathrm{e}^{-8s} & 8\mathrm{e}^{-8s}\\ 8\mathrm{e}^{-8s} & 8\mathrm{e}^{-8s}\end{pmatrix}$$

$$(N_{ij}(s))_{2\times2}=\begin{pmatrix}4\mathrm{e}^{-4s} & 4\mathrm{e}^{-4s}\\ 4\mathrm{e}^{-4s} & 4\mathrm{e}^{-4s}\end{pmatrix}\qquad (I_i)_{2\times1}=\begin{pmatrix}5\\ 5\end{pmatrix}\qquad (J_j)_{2\times1}=\begin{pmatrix}10\\ 10\end{pmatrix}$$

$$(f_i(\boldsymbol{u}))_{2\times1}=\begin{pmatrix}\frac{1}{2}\boldsymbol{u}\\ \frac{1}{4}\boldsymbol{u}\end{pmatrix}\qquad (\gamma_{ij})_{2\times1}=\begin{pmatrix}1\\ 1\end{pmatrix}\qquad (g_j(\boldsymbol{u}))_{2\times1}=\begin{pmatrix}\frac{1}{4}\boldsymbol{u}+1\\ \frac{1}{3}\boldsymbol{u}+3\end{pmatrix}\qquad (\tau_{ji})=\begin{pmatrix}1\\ 1\end{pmatrix}$$

当 $T=\mathbb{R}$ 或 $\mathbb{Z}$ 时，系统(29)的平衡点 $(\boldsymbol{u}^*,\boldsymbol{v}^*)=(u_1^*,u_2^*,\nu_1^*,\nu_2^*)^{\mathrm{T}}$ 是全局指数稳定的.

证 通过计算我们得到：

$$c_1-F_1\sum_{j=1}^{2}(|h_{1j}|+|h_{1j}^{*}|)\approx0.31>0\qquad c_2-F_2\sum_{j=1}^{2}(|h_{2j}|+|h_{2j}^{*}|)\approx0.28>0$$

$$d_1-G_1\sum_{i=1}^{2}(|\omega_{1i}|+|\omega_{1i}^{*}|)\approx0.29>0\qquad d_2-G_2\sum_{i=1}^{2}(|\omega_{2i}|+|\omega_{2i}^{*}|)\approx0.72>0$$

显然条件(H_2)和(H_3)是满足的.

1) 当 $T=\mathbb{R}$ 时，$\sigma(t)=t$，$\mu(t)=0$，我们得到

$$-2\sum_{k=1}^{2}D_{1k}-2c_1+\sum_{j=1}^{2}|\omega_{j1}|G_j+\sum_{j=1}^{2}|h_{1j}|F_1+2\sum_{j=1}^{2}|\omega_{j1}^{*}|G_j+2\mathrm{e}^2\sum_{j=1}^{2}|h_{1j}^{*}|F_i\approx-1.54<0$$

$$-2\sum_{k=1}^{2}D_{2k}-2c_2+\sum_{j=1}^{2}|\omega_{j2}|G_j+\sum_{j=1}^{2}|h_{2j}|F_2+2\sum_{j=1}^{2}|\omega_{j2}^{*}|G_j+2\mathrm{e}^2\sum_{j=1}^{2}|h_{2j}^{*}|F_2\approx-1.57<0$$

$$-2\sum_{k=1}^{2}D_{1k}^{*}-2d_1+\sum_{i=1}^{2}|h_{i1}|F_i+\sum_{i=1}^{2}|\omega_{1i}|G_1+2\sum_{i=1}^{2}|h_{i1}^{*}|F_i+2\mathrm{e}^2\sum_{i=1}^{2}|\omega_{1i}^{*}|G_1\approx-1.55<0$$

$$-2\sum_{k=1}^{2}D_{2k}^{*}-2d_2+\sum_{i=1}^{2}|h_{i2}|F_i+\sum_{i=1}^{2}|\omega_{2i}|G_2+2\sum_{i=1}^{2}|h_{i2}^{*}|F_i+2\mathrm{e}^2\sum_{i=1}^{2}|\omega_{2i}^{*}|G_2\approx-2.49<0$$

因此条件(H_1)－(H_5)成立，由定理1可知，系统(29)的平衡点$(\boldsymbol{u}^*, \boldsymbol{v}^*)=(u_1^*, u_2^*, \nu_1^*, \nu_2^*)^{\mathrm{T}}$是全局指数稳定的.

2）当 $T=\mathbb{Z}$ 时，$\sigma(t)=t+1$，$\mu(t)=1$，我们得到

$$-2\sum_{k=1}^{2}D_{1k}-2c_1+\sum_{j=1}^{2}|\omega_{j1}|G_j+\sum_{j=1}^{2}|h_{1j}|F_1+2\sum_{j=1}^{2}|\omega_{j1}^*|G_j+8\sum_{j=1}^{2}|h_{1j}^*|F_i\approx-2<0$$

$$-2\sum_{k=1}^{2}D_{2k}-2c_2+\sum_{j=1}^{2}|\omega_{j2}|G_j+\sum_{j=1}^{2}|h_{2j}|F_2+2\sum_{j=1}^{2}|\omega_{j2}^*|G_j+8\sum_{j=1}^{2}|h_{2j}^*|F_2\approx-1.8<0$$

$$-2\sum_{k=1}^{2}D_{1k}^*-2d_1+\sum_{i=1}^{2}|h_{i1}|F_i+\sum_{i=1}^{2}|\omega_{1i}|G_1+2\sum_{i=1}^{2}|h_{i1}^*|F_i+8\sum_{i=1}^{2}|\omega_{1i}^*|G_1\approx-1.72<0$$

$$-2\sum_{k=1}^{2}D_{2k}^*-2d_2+\sum_{i=1}^{2}|h_{i2}|F_i+\sum_{i=1}^{2}|\omega_{2i}|G_2+2\sum_{i=1}^{2}|h_{i2}^*|F_i+2\mathrm{e}^2\sum_{i=1}^{2}|\omega_{2i}^*|G_2\approx-2.71<0$$

因此条件(H_1)－(H_5)成立，由定理1可知，系统(29)的平衡点$(\boldsymbol{u}^*, \boldsymbol{v}^*)=(u_1^*, u_2^*, \nu_1^*, \nu_2^*)^{\mathrm{T}}$是全局指数稳定的.

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Stability Analysis for BAM Neural Networks with Reaction-Diffusion Terms and Dirichlet Boundary Conditions on Time Scales

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Abstract: The stability of equilibrium solution to BAM cellular neural networks with Dirichlet boundary conditions and reaction-diffusion terms on time scales is studied. Using Lyapunov functional and inequality skills, we find some sufficient conditions ensuring global exponential stability of equilibrium point for BAM cellular neural networks with Dirichlet boundary conditions and reaction-diffusion terms on time scales. An example is given to show the effectiveness of the results obtained.

Key words: BAM neural networks; Dirichlet boundary value; reaction-diffusion; exponential stability; time scale

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