DOI: 10.13718/j.cnki.xdzk.2016.08.011

G_{pb}-度量空间中弱压缩映射的公共不动点定理[①]

卢　瑶[1]，　邓　磊[1]，　杨明歌[2]

1. 西南大学 数学与统计学院，重庆 400715；2. 洛阳师范学院 数学科学学院，河南 洛阳 471022

摘要：在 G_p -度量和 G_b -度量的基础上，定义了 G_{pb} -度量空间，并在一定条件下，证明了弱压缩映射的公共不动点定理，该结果改进和推广了近期的相关结果.

关　键　词：G_{pb} -度量空间；弱压缩映射；公共不动点

中图分类号：O177.91　　**文献标志码**：A　　**文章编号**：1673-9868(2016)08-0061-06

2006 年，文献[1]首次定义了 G -度量空间. 2011 年，文献[2]结合偏度量和 G -度量，定义了 G_p -度量空间. 2012 年，文献[3]结合 G -度量和 b -度量，定义了 G_b -度量空间. 2015 年，文献[4]在 G_p -度量空间中证明了弱压缩映射的公共不动点定理. 本文在 G_p -度量和 G_b -度量的基础上，定义了 G_{pb} -度量空间，并在该空间中证明了相应弱压缩映射的公共不动点定理，该结果改进和推广了文献[4]中的相关结果.

1　预备知识

用 $\mathbb{N}$ 表示自然数集.

定义 1[1]　令 X 为非空集合，$G: X\times X\times X \longrightarrow [0, \infty)$，若满足以下条件：

(a) $x=y=z \Rightarrow G(x, y, z)=0$；

(b) $\forall x, y, z\in X$，$x\neq y$，有 $G(x, y, z)>0$；

(c) $\forall x, y, z\in X$，$y\neq z$，有 $G(x, x, y)\leqslant G(x, y, z)$；

(d) $G(x, y, z)=G(x, z, y)=G(y, z, x)=\cdots$；

(e) $\forall x, y, z, a\in X$，有 $G(x, y, z)\leqslant G(x, a, a)+G(a, y, z)$.

则称 $G(X, G)$ 为 G -度量空间.

定义 2[2]　令 X 为非空集合，$G_p: X\times X\times X \longrightarrow [0, \infty)$，若满足以下条件：

(a) $G_p(x, x, x)=G_p(y, y, y)=G_p(z, z, z) \Rightarrow x=y=z$；

(b) $\forall x, y, z\in X$，$x\neq y$，有 $0\leqslant G_p(x, x, x)\leqslant G_p(x, x, y)\leqslant G_p(x, y, z)$；

(c) $G_p(x, y, z)=G_p(x, z, y)=G_p(y, z, x)=\cdots$；

(d) $\forall x, y, z, a\in X$，有 $G_p(x, y, z)\leqslant G_p(x, a, a)+G_p(a, y, z)-G_p(a, a, a)$.

则称 $G(X, G_p)$ 为 G_p -度量空间.

定义 3[3]　令 X 为非空集合，$G_b: X\times X\times X \longrightarrow [0, \infty)$，若满足以下条件：

(a) $x=y=z \Rightarrow G_b(x, y, z)=0$；

(b) $\forall x, y\in X$，$x\neq y$，有 $G_b(x, x, y)>0$；

(c) $\forall x, y, z\in X$，$y\neq z$，有 $G_b(x, x, y)\leqslant G_b(x, y, z)$；

---

① 收稿日期：2015-11-10

基金项目：国家自然科学基金项目(11226228)；河南省高等学校重点科研项目(15A110036).

作者简介：卢　瑶(1992-)，女，重庆涪陵人，硕士研究生，主要从事非线性泛函分析的研究.

(d) $G_b(x, y, z) = G_b(x, z, y) = G_b(y, z, x) = \cdots$;

(e) $\forall x, y, z, a \in X$，有 $G_b(x, y, z) \leqslant s[G_b(x, a, a) + G_b(a, y, z)]$.

则称 $G(X, G_b)$ 为 G_b-度量空间.

定义 4 令 X 为非空集合，$G_{pb}: X \times X \times X \longrightarrow [0, \infty)$，$s \geqslant 1$，若满足以下条件：

(a) $G_{pb}(x, x, x) = G_{pb}(y, y, y) = G_{pb}(z, z, z) \Rightarrow x = y = z$;

(b) $\forall x, y, z \in X$，$x \neq y$，有 $0 \leqslant G_{pb}(x, x, x) \leqslant G_{pb}(x, x, y) \leqslant G_{pb}(x, y, z)$;

(c) $G_{pb}(x, y, z) = G_{pb}(x, z, y) = G_{pb}(y, z, x) = \cdots$;

(d) $\forall x, y, z, a \in X$，有 $G_{pb}(x, y, z) \leqslant s[G_{pb}(x, a, a) + G_{pb}(a, y, z)] - G_{pb}(a, a, a)$.

则称(X, G_{pb})为 G_{pb}-度量空间.

注 1 G_p-度量空间一定为 G_{pb}-度量空间，但反之不成立.

例 1 令 $X = [1, \infty)$，$G_{pb}(x, y, z) = \ln xyz$，$s \geqslant 1$，则 X 为 G_{pb}-度量空间，而不是 G_p-度量空间.

证 由 $G_{pb}(x, x, x) = G_{pb}(y, y, y) = G_{pb}(z, z, z)$，得 $\ln x^3 = \ln y^3 = \ln z^3$，即 $x = y = z$，则定义 4 的条件(a)成立.

对 $\forall x, y, z \in X$，不失一般性，设 $x \leqslant y \leqslant z$，则 $\ln x \leqslant \ln y \leqslant \ln z$，因而

$$3\ln x \leqslant 2\ln x + \ln y \leqslant \ln x + \ln y + \ln z$$

即 $\ln x^3 \leqslant \ln x^2 y \leqslant \ln xyz$，也即 $G_{pb}(x, x, x) \leqslant G_{pb}(x, x, y) \leqslant G_{pb}(x, y, z)$，则定义 4 的条件(b)成立.

又因 $\ln xyz = \ln xzy = \ln yzx = \cdots$，则 $G_{pb}(x, y, z) = G_{pb}(x, z, y) = G_{pb}(y, z, x) = \cdots$，则定义 4 的条件(c)成立.

又由 $s \geqslant 1$，$a \in X$，则

$$\begin{aligned} G_{pb}(x, y, z) = \ln xyz = \ln x + \ln y + \ln z \leqslant \\ s(\ln x + \ln y + \ln z) \leqslant s(\ln x + \ln y + \ln z) + 3\ln a(s-1) = \\ s(\ln x + \ln a + \ln a + \ln a + \ln y + \ln z) - 3\ln a = \\ s(\ln xa^2 + \ln ayz) - \ln a^3 = \\ s[G_{pb}(x, a, a) + G_{pb}(a, y, z)] - G_{pb}(a, a, a) \end{aligned}$$

则定义 4 的条件(d)成立. 因此，X 为一个 G_{pb}-度量空间，而不是 G_p-度量空间.

定义 5 令 $G(X, G_{pb})$ 为 G_{pb}-度量空间，$\{x_n\} \subset X$，$x \in X$，若 $\lim\limits_{n,m\to\infty} G_{pb}(x, x_n, x_m) = G_{pb}(x, x, x)$，则称 $x_n \to x$.

定义 6 令 $G(X, G_{pb})$ 为 G_{pb}-度量空间，称：

(i) $\{x_n\}$ 为 G_{pb}-柯西列当且仅当 $\lim\limits_{n,m\to\infty} G_{pb}(x_n, x_m, x_m)$ 存在且有限；

(ii) (X, G_{pb}) 为 G_{pb}-完备空间当且仅当任意 G_{pb}-柯西列都收敛，且 $\lim\limits_{n,m\to\infty} G_{pb}(x_n, x_m, x_m) = G_{pb}(x, x, x)$.

引理 1 令 $G(X, G_{pb})$ 为 G_{pb}-度量空间，则：

(i) 若 $G_{pb}(x, y, z) = 0$，则 $x = y = z$；

(ii) 若 $x \neq y$，则 $G_{pb}(x, y, y) > 0$.

证 (i) 假设 $x \neq y \neq z$，则有 $G_{pb}(x, x, x) < G_{pb}(x, y, z) = 0$，这与 $G_{pb}(x, x, x) \geqslant 0$ 矛盾，因而 $x = y = z$.

(ii) 假设 $G_{pb}(x, y, y) = 0$，则由(i)可得 $x = y$，这与 $x \neq y$ 矛盾，因此 $G_{pb}(x, y, y) > 0$.

定义 7[4] 定义两个函数集：

$$\Psi = \{\psi \mid \psi: [0, \infty) \longrightarrow [0, \infty),\ \psi\text{ 连续，非减，且 }\psi^{-1}(0) = 0\}$$

$$\Phi = \{\varphi \mid \varphi: [0, \infty) \longrightarrow [0, \infty),\ \varphi\text{ 下半连续，非减，且 }\varphi^{-1}(0) = 0\}$$

定义 8[5] $(X, \preccurlyeq)$ 为一个偏序集，映射 $f, g: X \longrightarrow X$，若对 $\forall x \in X$，有 $fx \preccurlyeq gfx$，$gx \preccurlyeq fgx$，则称 f, g 为弱增映射.

2　主要结果

在这部分，将在 G_{pb} -度量空间中建立两个弱增映射的公共不动点定理.

定理 1　令$(X,\leqslant)$为一个偏序集，f,g 为弱增映射，(X,G_{pb})为 G_{pb} -完备的度量空间，$s\geqslant 1$，$\psi\in\Psi$，$\varphi\in\Phi$，$\sum_{n=0}^{\infty} s^{m-n}G_{pb}(x_n,x_{n+1},x_{n+1})<\infty$，使得

$$\psi(G_{pb}(fx,gy,gy))\leqslant\psi(M(x,y,y))-\varphi(M(x,y,y)) \tag{1}$$

其中

$$M(x,y,y)=$$
$$\max\{G_{pb}(x,y,y),G_{pb}(x,fx,fx),G_{pb}(y,gy,gy),[G_{pb}(x,gy,gy)+G_{pb}(y,fx,fx)]/2s\}$$

若以下条件之一成立：

(i) f 连续或 g 连续；

(ii) $\{x_n\}$ 非减，$z\in X$，$x_n\rightarrow z\Rightarrow x_n\leqslant z$.

则 f,g 有公共不动点.

证　先证：只要 f,g 中一个有不动点，则该点一定为 f,g 的公共不动点. 不失一般性，假设 u 为 f 的不动点，即 $u=fu$，由(1) 式，令 $x=y=u$，则

$$\begin{aligned}\psi(G_{pb}(u,gu,gu))=\psi(G_{pb}(fu,gu,gu))\leqslant\\ \psi(M(u,u,u))-\varphi(M(u,u,u))\end{aligned} \tag{2}$$

其中

$$\begin{aligned}M(u,u,u)=&\max\{G_{pb}(u,u,u),G_{pb}(u,fu,fu),G_{pb}(u,gu,gu),\\&[G_{pb}(u,gu,gu)+G_{pb}(u,fu,fu)]/2s\}=\\&\max\{G_{pb}(u,u,u),G_{pb}(u,gu,gu)\}=\\&G_{pb}(u,gu,gu)\end{aligned} \tag{3}$$

将(3) 式代入(2) 式，有

$$\psi(G_{pb}(u,gu,gu))\leqslant\psi(G_{pb}(u,gu,gu))-\varphi(G_{pb}(u,gu,gu)) \tag{4}$$

则 $\varphi(G_{pb}(u,gu,gu))\leqslant 0$，因而 $G_{pb}(u,gu,gu)=0$，即 $u=gu$. 因此可得 $fu=u=gu$，即 u 为 f,g 的公共不动点.

下证 f 或 g 有不动点. 任取 $x_0\in X$，若 $fx_0=x_0$，则结论成立. 设 $fx_0\neq x_0$，则可构造如下序列：

$$x_1=fx_0\leqslant gfx_0=gx_1=x_2$$
$$x_2=gx_1\leqslant fgx_1=fx_2=x_3$$
$$\vdots$$
$$x_n\leqslant x_{n+1}$$

以下分两种情况：

情况 1　当 $G_{pb}(x_{2n},x_{2n+1},x_{2n+1})=0$ 时，即 $x_{2n}=x_{2n+1}$ 时，由(1) 式有

$$\begin{aligned}\psi(G_{pb}(x_{2n},x_{2n+1},x_{2n+1}))=\psi(G_{pb}(fx_{2n},gx_{2n+1},gx_{2n+1}))\leqslant\\ \psi(M(x_{2n},x_{2n+1},x_{2n+1}))-\varphi(M(x_{2n},x_{2n+1},x_{2n+1}))\end{aligned} \tag{5}$$

其中

$$\begin{aligned}M(x_{2n},x_{2n+1},x_{2n+1})=&\max\{G_{pb}(x_{2n},x_{2n+1},x_{2n+1}),G_{pb}(x_{2n},fx_{2n},fx_{2n}),G_{pb}(x_{2n+1},gx_{2n+1},gx_{2n+1}),\\&[G_{pb}(x_{2n},gx_{2n+1},gx_{2n+1})+G_{pb}(x_{2n+1},fx_{2n},fx_{2n})]/2s\}=\\&\max\{G_{pb}(x_{2n},x_{2n+1},x_{2n+1}),G_{pb}(x_{2n},x_{2n+1},x_{2n+1}),G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}),\\&[G_{pb}(x_{2n},x_{2n+2},x_{2n+2})+G_{pb}(x_{2n+1},x_{2n+1},x_{2n+1})]/2s\}\leqslant\\&\max\{G_{pb}(x_{2n},x_{2n+1},x_{2n+1}),G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}),\\&[s(G_{pb}(x_{2n},x_{2n+1},x_{2n+1})+G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))-\\&G_{pb}(x_{2n+1},x_{2n+1},x_{2n+1})+G_{pb}(x_{2n+1},x_{2n+1},x_{2n+1})]/2s\}\end{aligned} \tag{6}$$

因为 $x_{2n}=x_{2n+1}$，则 $M(x_{2n},x_{2n+1},x_{2n+1})=G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2})$，将其代入(5)式，有

$$\psi(G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))\leqslant\psi(G_{pb}(x_{2n},x_{2n+1},x_{2n+1}))-\varphi(G_{pb}(x_{2n},x_{2n+1},x_{2n+1}))$$

则 $\varphi(G_{pb}(x_{2n},x_{2n+1},x_{2n+1}))\leqslant 0$，因而

$$G_{pb}(x_{2n},x_{2n+1},x_{2n+1})=0$$

即 $x_{2n+1}=x_{2n+2}$. 同理可得

$$x_{2n+2}=x_{2n+3}$$

所以 x_{2n} 为 f,g 的公共不动点.

情况 2　当 $G_{pb}(x_{2n},x_{2n+1},x_{2n+1})>0$ 时，则(6)式变为

$$M(x_{2n},x_{2n+1},x_{2n+1})=\max\{G_{pb}(x_{2n},x_{2n+1},x_{2n+1}),G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2})\}$$

若

$$G_{pb}(x_{2n},x_{2n+1},x_{2n+1})\leqslant G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2})$$

则

$$M(x_{2n},x_{2n+1},x_{2n+1})=G_{pb}(x_{2n},x_{2n+1},x_{2n+1})$$

将其代入(5)式，有

$$\psi(G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))\leqslant\psi(G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))-\varphi(G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))$$

则

$$\varphi(G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2}))\leqslant 0$$

得

$$G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2})=0$$

所以

$$G_{pb}(x_{2n},x_{2n+1},x_{2n+1})=0$$

这与 $G_{pb}(x_{2n},x_{2n+1},x_{2n+1})>0$ 矛盾，因此有

$$M(x_{2n},x_{2n+1},x_{2n+1})=G_{pb}(x_{2n},x_{2n+1},x_{2n+1})$$

即

$$G_{pb}(x_{2n},x_{2n+1},x_{2n+1})\geqslant G_{pb}(x_{2n+1},x_{2n+2},x_{2n+2})$$

同理可得

$$G_{pb}(x_{2n-1},x_{2n},x_{2n})\geqslant G_{pb}(x_{2n},x_{2n+1},x_{2n+1})$$

因此有

$$G_{pb}(x_n,x_{n+1},x_{n+1})\geqslant G_{pb}(x_{n+1},x_{n+2},x_{n+2})$$

即 $\{G_{pb}(x_{n+1},x_{n+2},x_{n+2})\}$ 为非增有界序列，则存在 $L\geqslant 0$，使得

$$\lim_{n\to\infty}G_{pb}(x_{n+1},x_{n+2},x_{n+2})=L$$

又因 φ 下半连续，则

$$\varphi(L)\leqslant\liminf_{n\to\infty}\varphi(M(x_n,x_{n+1},x_{n+1}))$$

由(1)式有

$$\psi(G_{pb}(x_{n+1},x_{n+2},x_{n+2}))\leqslant\psi(M(x_n,x_{n+1},x_{n+1}))-\varphi(M(x_n,x_{n+1},x_{n+1}))$$

对之取极限，有

$$\psi(L)\leqslant\psi(L)-\varphi(L)$$

则 $\varphi(L)=0$，即 $L=0$. 从而

$$\lim_{n\to\infty}G_{pb}(x_{n+1},x_{n+2},x_{n+2})=0$$

下证 $\{x_n\}$ 为 G_{pb}-柯西列. 对 $\forall n,m\in\mathbb{N}$，$n\leqslant m$，有

$$\begin{aligned}G_{pb}(x_n,x_m,x_m)\leqslant{}& s(G_{pb}(x_{m-1},x_m,x_m)+G_{pb}(x_n,x_{m-1},x_{m-1}))-G_{pb}(x_{m-1},x_{m-1},x_{m-1})\leqslant\\&sG_{pb}(x_{m-1},x_m,x_m)+s[s(G_{pb}(x_n,x_{m-2},x_{m-2})+G_{pb}(x_{m-2},x_{m-1},x_{m-1}))-\\&G_{pb}(x_{m-2},x_{m-2},x_{m-2})]-G_{pb}(x_{m-1},x_{m-1},x_{m-1})\leqslant\end{aligned}$$

$$sG_{pb}(x_{m-1}, x_m, x_m) + s^2 G_{pb}(x_{m-2}, x_{m-1}, x_{m-1}) + \cdots +$$
$$s^{m-n-2} G_{pb}(x_{n+2}, x_{n+3}, x_{n+3}) +$$
$$s^{m-n-1}[G_{pb}(x_n, x_{n+1}, x_{n+1}) + G_{pb}(x_{n+1}, x_{n+2}, x_{n+2})] =$$
$$\sum_{k=n+1}^{m-1} s^{m-k} G_{pb}(x_k, x_{k+1}, x_{k+1}) + s^{m-n-1} G_{pb}(x_n, x_{n+1}, x_{n+1}) =$$
$$\sum_{k=n}^{m-1} s^{m-k} G_{pb}(x_k, x_{k+1}, x_{k+1}) + s^{m-n-1} G_{pb}(x_n, x_{n+1}, x_{n+1}) - s^{m-n} G_{pb}(x_n, x_{n+1}, x_{n+1}) =$$
$$\sum_{k=n}^{m-1} s^{m-k} G_{pb}(x_k, x_{k+1}, x_{k+1}) - s^{m-n} G_{pb}(x_n, x_{n+1}, x_{n+1})(1 - 1/s) \leqslant$$
$$\sum_{k=n}^{m-1} s^{m-k} G_{pb}(x_k, x_{k+1}, x_{k+1})$$

令

$$Z_n(x) = \sum_{k=0}^{n} s^{m-k} G_{pb}(x_k, x_{k+1}, x_{k+1})$$

因为

$$\sum_{n=0}^{\infty} s^{m-k} G_{pb}(x_n, x_{n+1}, x_{n+1}) < \infty$$

则

$$\lim_{n,m\to\infty} G_{pb}(x_n, x_m, x_m) \leqslant \lim_{n,m\to\infty} (Z_{m-1}(x) - Z_{n-1}(x)) = 0$$

即

$$\lim_{n,m\to\infty} G_{pb}(x_n, x_m, x_m) = 0$$

则$\{x_n\}$为 G_{pb}-柯西列.

又因(X, G_{pb})为 G_{pb}-完备度量空间，则存在 $z \in X$，使得 $x_n \to z$. 以下分两种情况来证明：

若满足条件(i)时，不失一般性，设 g 连续. 因为 $x_{2n+2} = gx_{2n+1}$，则 $z = gz$，有 $gz = z = fz$. 所以 z 为 f, g 的公共不动点.

若满足条件(ii)时，由(1)式有

$$\psi(G_{pb}(x_{2n+1}, gz, gz)) = \psi(G_{pb}(fx_{2n}, gz, gz)) \leqslant \psi(M(x_{2n}, z, z)) - \varphi(M(x_{2n}, z, z)) \tag{7}$$

其中

$$\begin{aligned} M(x_{2n}, z, z) = &\max\{G_{pb}(x_{2n}, z, z), G_{pb}(x_{2n}, fx_{2n}, fx_{2n}), G_{pb}(z, gz, gz), \\ &[G_{pb}(x_{2n}, gz, gz) + G_{pb}(z, fx_{2n}, fx_{2n})]/2s\} = \\ &\max\{G_{pb}(x_{2n}, z, z), G_{pb}(x_{2n}, x_{2n+1}, x_{2n+1}), G_{pb}(z, gz, gz), \\ &[G_{pb}(x_{2n}, gz, gz) + G_{pb}(z, x_{2n+1}, x_{2n+1})]/2s\} \end{aligned} \tag{8}$$

对(8)式取极限有

$$\lim_{n\to\infty} M(x_{2n}, z, z) = G_{pb}(z, gz, gz)$$

代入(7)式，有

$$\begin{aligned} \psi(G_{pb}(z, gz, gz)) = &\limsup_{n\to\infty} \psi(fx_{2n}, gz, gz) \leqslant \\ &\limsup_{n\to\infty} [\psi(M(x_{2n}, z, z)) - \varphi(M(x_{2n}, z, z))] \leqslant \\ &\psi(G_{pb}(z, gz, gz)) - \varphi(G_{pb}(z, gz, gz)) \end{aligned}$$

则

$$\varphi(G_{pb}(z, gz, gz)) \leqslant 0$$

即

$$G_{pb}(z, gz, gz) = 0$$

也即 $gz=z$. 因此，$gz=z=fz$.

所以 z 为 f,g 的公共不动点.

注 2 当 $s=1$ 时，定理 1 则为文献[4] 中相关的结果.

推论 1 令$(X,\preccurlyeq)$为一个偏序集，f,g 为弱增映射，(X,G_{pb})为 G_{pb} -完备的度量空间，$s\geqslant 1$，$\psi\in\Psi$，$\varphi\in\Phi$，$\sum\limits_{n=0}^{\infty} s^{m-n}G_{pb}(x_n,x_{n+1},x_{n+1})<\infty$，使得

$$\psi(G_{pb}(fx,gy,gy))\leqslant M(x,y,y)-\varphi(M(x,y,y))$$

其中

$$M(x,y,y)=\max\{G_{pb}(x,y,y),G_{pb}(x,fx,fx),G_{pb}(y,gy,gy),\\ [G_{pb}(x,gy,gy)+G_{pb}(y,fx,fx)]/2s\}$$

若以下条件之一成立：

(ⅰ) f 连续或 g 连续；

(ⅱ) $\{x_n\}$ 非减，$z\in X$，$x_n\to z\Rightarrow x_n\preccurlyeq z$.

则 f,g 有公共不动点.

参考文献：

[1] MUSTAFA Z, SIMS B. A New Approach to Generalized Metric Spaces [J]. J Nonlinear Convex Anl, 2006, 7(7): 289－297.

[2] ZAND M R A, NEZHAD A D. A Generalization of Partial Metric Spaces [J]. J Contemp Appl Math, 2011, 1(1): 86－93.

[3] AGHAJANI A, ABBAS M, ROSHAN J R. Common Fixed Point of Generalized Weak Contractive Mappings in Partially Ordered B-Metric Spaces [J]. Applied Math & Computation, 2012, 218(218): 9383－9395.

[4] BARAKAT M A, ZIDAN A M. A Common Fixed Point Theorem for Weak Contractive Maps in G_p-Metric Spaces [J]. Journal of Egyptian Mathematical Society, 2015, 23(2): 121－124.

[5] AHMED M A. Some Fixed Point Theorems [J]. Int J Geom Methods Modern Phys, 2011, 8(1): 17－21.

A Common Fixed Point Theorem for Weak Contraction Maps in G_{pb}-Metric Spaces

LU Yao[1], DENG Lei[1], YANG Ming-ge[2]

1. School of Mathematics and Statistics, Southwest University, Chongqing 400715, China;

2. College of Mathematics Science, Luoyang Normal University, Luoyang Henan 471022, China

Abstract: In this paper, we define a G_{pb}-metric space which is based on the G_p-metric and G_b-metric, and prove the common fixed point theorem for weak contraction maps in this space. As a result, we obtain some related fixed point theorems, which largely improve and extend some related results that have been published recently in G_p-metric space.

Key words: G_{pb}-metric space; weak contraction maps; common fixed point

责任编辑 廖 坤