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一类反周期函数缺项插值问题的解[①]

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摘要：对一类反周期函数双周期插值问题进行分析，根据插值条件，结合基多项式的分解定理，建立相应的线性方程组，由克莱默法则给出插值问题有解的充分必要条件，并得到该条件下插值解的显式表达式.

关 键 词：反周期函数；双周期；插值

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三角函数的插值问题是函数逼近论的一个重要的研究分支. 近年来，关于插值问题解的存在性与表示、插值问题的收敛与饱和等方面产生了很多好的结论[1-8]. 文献[4]提出了π-周期及其反周期三角多项式空间与基多项式的概念，并定义了 Sharma-Varma 算子：

$$A_n^m \mathrm{e}^{ikx} = a_n^m(k)\mathrm{e}^{ikx}$$

$$B_n^m \mathrm{e}^{ikx} = b_n^m(k)\mathrm{e}^{ikx}$$

其中：

$$a_n^m(k) \overset{\wedge}{=} a_n^m = \frac{[i(k+n)]^m + [i(k-n)]^m}{2}$$

$$b_n^m(k) \overset{\wedge}{=} b_n^m = \frac{[i(k+n)]^m - [i(k-n)]^m}{2i}$$

本文所用其它记号与文献[4]中的完全一致，在此不做叙述. 需要指出的一个结果是：

对于给定的结点组：

$$x_k = \frac{k\pi}{n} \qquad k = 0, 1, \cdots, n-1$$

当 n 为偶数时，$\{L_n(x - x_k) \mid k = 0, \cdots, n-1\}$ 是 $\omega_{n-1}^{\perp}$ 的一组基，其中

$$L_n(x) = \frac{2}{n}\sum_{l=1}^{\frac{n}{2}} \cos\,(2l-1)x$$

当 n 为奇数时，$\{L_n(x - x_k) \mid k = 0, \cdots, n-1\}$ 是 ω_{n-1} 的一组基，其中

$$L_n(x) = \frac{1}{n} + \frac{2}{n}\sum_{l=1}^{\frac{n-1}{2}} \cos 2lx$$

文献[6]讨论了一类反周期函数的双周期插值问题，即是否存在 π-反周期的三角多项式 $T(x)$，满足：

$$T(x_{2k}) = \alpha_{0,k} \qquad T^{(m_1)}(x_{2k+1}) = \alpha_{1,k} \qquad T^{(m_2)}(x_{2k+1}) = \alpha_{2,k} \qquad T^{(m_3)}(x_{2k+1}) = \alpha_{3,k} \tag{1}$$

其中：$m_1 < m_2 < m_3$ 为正整数；$x_{2k} = \dfrac{2k\pi}{2n}$，$x_{2k+1} = \dfrac{2k+1}{2n}\pi$；$\{\alpha_{j,k}\}$ 为给定复数列；$k = 0, \cdots, n-1$，

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$j=0,\cdots,3$.

文献[8]通过改变结点组的位置考虑类似的插值问题，得到的结论是：插值问题有解的充要条件并未因为结点组的改变而发生变化，只是插值解的形式发生了改变. 本文受文献[8]的启发，考虑这样的插值问题：是否存在 π-反周期的三角多项式 $T(x)$，满足条件：

$$T(x_{2k})=\alpha_{0,k}\qquad T^{(m_1)}(x_{2k+1})=\alpha_{1,k}\qquad T^{(m_2)}(x_{2k})=\alpha_{2,k}\qquad T^{(m_3)}(x_{2k+1})=\alpha_{3,k}\tag{2}$$

引理 1[6] 若 n 为偶数，$T(x)\in\omega_{4n-1}^{\perp}$，则存在 $U_j\in\omega_{n-1}^{\perp}$，$j=0,\cdots,3$，使得

$$T(x)=\frac{1}{2}\left[\cos 3nx\left(U_0-\frac{1}{\sqrt{2}}U_1+\frac{1}{\sqrt{2}}U_3\right)+\sin 3nx\left(\frac{1}{\sqrt{2}}U_1-U_2+\frac{1}{\sqrt{2}}U_3\right)\right]+$$
$$\frac{1}{2}\left[\cos nx\left(U_0+\frac{1}{\sqrt{2}}U_1-\frac{1}{\sqrt{2}}U_3\right)+\sin nx\left(\frac{1}{\sqrt{2}}U_1+U_2+\frac{1}{\sqrt{2}}U_3\right)\right]\tag{3}$$

若 n 为奇数，则存在 $U_j\in\omega_{n-1}(j=0,\cdots,3)$，使得(3)式仍然成立.
其中

$$U_j=U_j(x)=\sum_{k=0}^{n-1}(-1)^k T(z_{4k+j})L_n(x-z_{4k+j})\qquad z_k=\frac{k\pi}{4n};\ k=0,1,\cdots,4n-1;\ j=0,1,2,3$$

为了符号简洁，我们记：

$$V_1=\frac{1}{2}\left(U_0-\frac{1}{\sqrt{2}}U_1+\frac{1}{\sqrt{2}}U_3\right)\qquad V_2=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_1-U_2+\frac{1}{\sqrt{2}}U_3\right)$$
$$V_3=\frac{1}{2}\left(U_0+\frac{1}{\sqrt{2}}U_1-\frac{1}{\sqrt{2}}U_3\right)\qquad V_4=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_1+U_2+\frac{1}{\sqrt{2}}U_3\right)$$

对任意的 $T(x)\in\omega_{4n-1}^{\perp}$，有

$$T(x)=\cos 3nxV_1+\sin 3nxV_2+\cos nxV_3+\sin nxV_4\tag{4}$$

显然有以下结果成立：

$$V_1+V_3=U_0=\sum_{k=0}^{n-1}(-1)^k T(z_{4k})L_n(x-z_{4k})=\sum_{k=0}^{n-1}(-1)^k T(x_{2k})L_n(x-x_{2k})\tag{5}$$

$$V_4-V_2=U_2=\sum_{k=0}^{n-1}(-1)^k T(z_{4k+2})L_n(x-z_{4k+2})=\sum_{k=0}^{n-1}(-1)^k T(x_{2k+1})L_n(x-x_{2k+1})\tag{6}$$

引理 2[8] 若 $T(x)\in\omega_{4n-1}^{\perp}$，则

$$D^m T(x)=\cos 3nx(A_{3n}^m V_1+B_{3n}^m V_2)+\sin 3nx(A_{3n}^m V_2-B_{3n}^m V_1)+$$
$$\cos nx(A_n^m V_3+B_n^m V_4)+\sin nx(A_n^m V_4-B_n^m V_3)$$

其中 m 为正整数，D^m 表示 m 阶微分算子.

定理 1 若 $m_1<m_2<m_3$ 为正整数，则存在 $T(x)\in\omega_{4n-1}^{\perp}$ 满足(2)式当且仅当 m_2 为偶数且 m_1,m_3 为一奇一偶. 且当该条件成立时，有

$$T(x)=\sum_{k=0}^{n-1}\left[\alpha_{0,k}T_0(x-x_{2k})+\alpha_{1,k}T_1(x-x_{2k+1})+\alpha_{2,k}T_2(x-x_{2k})+\alpha_{3,k}T_3(x-x_{2k+1})\right]$$

其中：

$$T_j(x)=\cos 3nxV_{j1}+\sin 3nxV_{j2}+\cos nxV_{j3}+\sin nxV_{j4}$$

$$V_{j1}=\frac{1}{2}\left(U_{j0}-\frac{1}{\sqrt{2}}U_{j1}+\frac{1}{\sqrt{2}}U_{j3}\right)$$

$$V_{j2}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{j1}-U_{j2}+\frac{1}{\sqrt{2}}U_{j3}\right)$$

$$V_{j3}=\frac{1}{2}\left(U_{j0}+\frac{1}{\sqrt{2}}U_{j1}-\frac{1}{\sqrt{2}}U_{j3}\right)$$

$$V_{j4}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{j1}+U_{j2}+\frac{1}{\sqrt{2}}U_{j3}\right)$$

$$U_{ji}=\frac{D_{ji}}{D}$$

$$D=\begin{vmatrix} 1 & 0 & 0 & 0 \\ \dfrac{b_{3n}^{m_1}-b_{n}^{m_1}}{2} & \dfrac{a_{n}^{m_1}-a_{3n}^{m_1}-b_{n}^{m_1}-b_{3n}^{m_1}}{2\sqrt{2}} & \dfrac{a_{n}^{m_1}+a_{3n}^{m_1}}{2} & \dfrac{a_{n}^{m_1}-a_{3n}^{m_1}+b_{n}^{m_1}+b_{3n}^{m_1}}{2\sqrt{2}} \\ \dfrac{a_{n}^{m_2}+a_{3n}^{m_2}}{2} & \dfrac{a_{n}^{m_2}-a_{3n}^{m_2}+b_{n}^{m_2}+b_{3n}^{m_2}}{2\sqrt{2}} & \dfrac{b_{n}^{m_2}-b_{3n}^{m_2}}{2} & \dfrac{a_{n}^{m_2}-a_{3n}^{m_2}+b_{n}^{m_2}+b_{3n}^{m_2}}{2\sqrt{2}} \\ \dfrac{b_{3n}^{m_3}-b_{n}^{m_3}}{2} & \dfrac{a_{n}^{m_3}-a_{3n}^{m_3}-b_{n}^{m_3}-b_{3n}^{m_3}}{2\sqrt{2}} & \dfrac{a_{n}^{m_3}+a_{3n}^{m_3}}{2} & \dfrac{a_{n}^{m_3}-a_{3n}^{m_3}+b_{n}^{m_3}+b_{3n}^{m_3}}{2\sqrt{2}} \end{vmatrix}$$

D_{ji} 为分别用$(L_n(x),0,0,0)^{\mathrm{T}}$，$(0,L_n(x-x_1),0,0)^{\mathrm{T}}$，$(0,0,L_n(x),0)^{\mathrm{T}}$，$(0,0,0,L_n(x-x_1))^{\mathrm{T}}$ 代替 D 的第 $i+1$ 列所产生的行列式，$i=0,\cdots,3$，$j=0,\cdots,3$.

定理 1 的证明

若存在 $T(x)\in\omega_{4n-1}^{\perp}$ 有以下形式：

$$T(x)=\sum_{k=0}^{n-1}\left[\alpha_{0k}T_0(x-x_{2k})+\alpha_{1k}T_1(x-x_{2k+1})+\alpha_{2k}T_2(x-x_{2k})+\alpha_{3k}T_3(x-x_{2k+1})\right]$$

满足(2) 式，需有基函数 $T_0(x),T_1(x),T_2(x),T_3(x)\in\omega_{4n-1}^{\perp}$ 满足以下条件($k=0,\cdots,n-1$)：

$$T_0(x_{2k})=\delta_{0k},T_0^{(m_1)}(x_{2k+1})=0,T_0^{(m_2)}(x_{2k})=0,T_0^{(m_3)}(x_{2k+1})=0 \tag{7}$$

$$T_1(x_{2k})=0,T_1^{(m_1)}(x_{2k+1})=\delta_{0k},T_1^{(m_2)}(x_{2k})=0,T_1^{(m_3)}(x_{2k+1})=0 \tag{8}$$

$$T_2(x_{2k})=0,T_2^{(m_1)}(x_{2k+1})=0,T_2^{(m_2)}(x_{2k})=\delta_{0k},T_2^{(m_3)}(x_{2k+1})=0 \tag{9}$$

$$T_3(x_{2k})=0,T_3^{(m_1)}(x_{2k+1})=0,T_3^{(m_2)}(x_{2k})=0,T_3^{(m_3)}(x_{2k+1})=\delta_{0k} \tag{10}$$

先来考虑 $T_0(x)$，由引理 1 可设

$$T_0(x)=\cos 3nxV_{01}+\sin 3nxV_{02}+\cos nxV_{03}+\sin nxV_{04}$$

其中：

$$V_{01}=\frac{1}{2}\left(U_{00}-\frac{1}{\sqrt{2}}U_{01}+\frac{1}{\sqrt{2}}U_{03}\right)$$

$$V_{02}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{01}-U_{02}+\frac{1}{\sqrt{2}}U_{03}\right)$$

$$V_{03}=\frac{1}{2}\left(U_{00}+\frac{1}{\sqrt{2}}U_{01}-\frac{1}{\sqrt{2}}U_{03}\right)$$

$$V_{04}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{01}+U_{02}+\frac{1}{\sqrt{2}}U_{03}\right)$$

由(4) 式可得

$$U_{00}=\sum_{k=0}^{n-1}(-1)^kT_0(x_{2k})L_n(x-x_{2k})=L_n(x) \tag{11}$$

再利用引理 2 可得

$$T_0^{(m_2)}(x_{2k})=\cos 3nx_{2k}(A_{3n}^{m_2}V_{01}+B_{3n}^{m_2}V_{02})+\sin 3nx_{2k}(A_{3n}^{m_2}V_{02}-B_{3n}^{m_2}V_{01})+$$
$$\cos nx_{2k}(A_{n}^{m_2}V_{03}+B_{n}^{m_2}V_{04})+\sin nx_{2k}(A_{n}^{m_2}V_{04}-B_{n}^{m_2}V_{03})=0$$

则：

$$A_{3n}^{m_2}V_{01}+B_{3n}^{m_2}+A_{n}^{m_2}V_{03}+B_{n}^{m_2}V_{04}=0 \tag{12}$$

$$T_0^{(m_j)}(x_{2k+1})=\cos 3nx_{2k+1}(A_{3n}^{m_j}V_{01}+B_{3n}^{m_j}V_{02})+\sin 3nx_{2k+1}(A_{3n}^{m_j}V_{02}-B_{3n}^{m_j}V_{01})+$$
$$\cos nx_{2k+1}(A_{n}^{m_j}V_{03}+B_{n}^{m_j}V_{04})+\sin nx_{2k+1}(A_{n}^{m_j}V_{04}-B_{n}^{m_j}V_{03})=0$$

即

$$B_{3n}^{m_j}V_{01}-A_{3n}^{m_j}V_{02}-B_{n}^{m_j}V_{03}+A_{n}^{m_j}V_{04}=0\qquad j=1,3 \tag{13}$$

联立(11)式、(12)式与(13)式，得方程组

$$\begin{cases} U_{00}=L_n(x) \\ \dfrac{(b_{3n}^{m_1}-b_n^{m_1})U_{00}}{2}+\dfrac{(a_n^{m_1}-a_{3n}^{m_1}-b_n^{m_1}-b_{3n}^{m_1})U_{01}}{2\sqrt{2}}+\dfrac{(a_n^{m_1}+a_{3n}^{m_1})U_{02}}{2}+\dfrac{(a_n^{m_1}-a_{3n}^{m_1}+b_n^{m_1}+b_{3n}^{m_1})U_{03}}{2\sqrt{2}}=0 \\ \dfrac{(a_n^{m_2}+a_{3n}^{m_2})U_{00}}{2}+\dfrac{(a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2})U_{01}}{2\sqrt{2}}+\dfrac{(b_n^{m_2}-b_{3n}^{m_2})U_{02}}{2}+\dfrac{(a_{3n}^{m_2}-a_n^{m_2}+b_n^{m_2}+b_{3n}^{m_2})U_{03}}{2\sqrt{2}}=0 \\ \dfrac{(b_{3n}^{m_3}-b_n^{m_3})U_{00}}{2}+\dfrac{(a_n^{m_3}-a_{3n}^{m_3}-b_n^{m_3}-b_{3n}^{m_3})U_{01}}{2\sqrt{2}}+\dfrac{(a_n^{m_3}+a_{3n}^{m_3})U_{02}}{2}+\dfrac{(a_n^{m_3}-a_{3n}^{m_3}+b_n^{m_3}+b_{3n}^{m_3})U_{03}}{2\sqrt{2}}=0 \end{cases}$$

再来考虑 $T_1(x)$，同样设：

$$T_1(x)=\cos 3nxV_{11}+\sin 3nxV_{12}+\cos nxV_{13}+\sin nxV_{14}$$

$$V_{11}=\frac{1}{2}\left(U_{10}-\frac{1}{\sqrt{2}}U_{11}+\frac{1}{\sqrt{2}}U_{13}\right)$$

$$V_{12}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{11}-U_{12}+\frac{1}{\sqrt{2}}U_{13}\right)$$

$$V_{13}=\frac{1}{2}\left(U_{10}+\frac{1}{\sqrt{2}}U_{11}-\frac{1}{\sqrt{2}}U_{13}\right)$$

$$V_{14}=\frac{1}{2}\left(\frac{1}{\sqrt{2}}U_{11}+U_{12}+\frac{1}{\sqrt{2}}U_{13}\right)$$

由(8)式和以上的讨论，有：

$$U_{10}=0 \tag{14}$$

$$T_1^{(m_2)}(x_{2k})=\cos 3nx_{2k}(A_{3n}^{m_2}V_{11}+B_{3n}^{m_2}V_{12})+\sin 3nx_{2k}(A_{3n}^{m_2}V_{12}-B_{3n}^{m_2}V_{11})+$$
$$\cos nx_{2k}(A_n^{m_2}V_{13}+B_n^{m_2}V_{14})+\sin nx_{2k}(A_n^{m_2}V_{14}-B_n^{m_2}V_{13})=0$$

即：

$$A_{3n}^{m_2}V_{01}+B_{3n}^{m_2}V_{02}+A_n^{m_2}V_{03}+B_n^{m_2}V_{04}=0 \tag{15}$$

$$T_1^{(m_j)}(x)=\cos 3nx(A_{3n}^{m_j}V_{11}+B_{3n}^{m_j}V_{12})+\sin 3nx(A_{3n}^{m_j}V_{12}-B_{3n}^{m_j}V_{11})+$$
$$\cos nx(A_n^{m_j}V_{13}+B_n^{m_j}V_{14})+\sin nx(A_n^{m_j}V_{14}-B_n^{m_j}V_{13}) \qquad j=1,2,3$$

由(6)式可知

$$A_n^{m_j}V_{14}-B_n^{m_j}V_{13}-A_{3n}^{m_j}V_{12}+B_{3n}^{m_j}V_{11}=$$
$$\sum_{k=0}^{n-1}(-1)^kT_1^{(m_j)}(x_{2k+1})L_n(x-x_{2k+1})=\begin{cases}L_n(x-x_1) & j=1\\ 0 & j=3\end{cases} \tag{16}$$

联立(14)式、(15)式与(16)式，得方程组

$$\begin{cases} U_{10}=0 \\ \dfrac{(b_{3n}^{m_1}-b_n^{m_1})U_{10}}{2}+\dfrac{(a_n^{m_1}-a_{3n}^{m_1}-b_n^{m_1}-b_{3n}^{m_1})U_{11}}{2\sqrt{2}}+\dfrac{(a_n^{m_1}+a_{3n}^{m_1})U_{12}}{2}+\dfrac{(a_n^{m_1}-a_{3n}^{m_1}+b_n^{m_1}+b_{3n}^{m_1})U_{13}}{2\sqrt{2}}=L_n(x-x_1) \\ \dfrac{(a_n^{m_2}+a_{3n}^{m_2})U_{10}}{2}+\dfrac{(a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2})U_{11}}{2\sqrt{2}}+\dfrac{(b_n^{m_2}-b_{3n}^{m_2})U_{12}}{2}+\dfrac{(a_{3n}^{m_2}-a_n^{m_2}+b_n^{m_2}+b_{3n}^{m_2})U_{13}}{2\sqrt{2}}=0 \\ \dfrac{(b_{3n}^{m_3}-b_n^{m_3})U_{10}}{2}+\dfrac{(a_n^{m_3}-a_{3n}^{m_3}-b_n^{m_3}-b_{3n}^{m_3})U_{11}}{2\sqrt{2}}+\dfrac{(a_n^{m_3}+a_{3n}^{m_3})U_{12}}{2}+\dfrac{(a_n^{m_3}-a_{3n}^{m_3}+b_n^{m_3}+b_{3n}^{m_3})U_{13}}{2\sqrt{2}}=0 \end{cases}$$

用类似的方法讨论 $T_2(x)$，$T_3(x)$，同样可得相应的方程组，而且 4 个方程组有相同的系数行列式：

$$D=\begin{vmatrix} 1 & 0 & 0 & 0 \\ \dfrac{b_{3n}^{m_1}-b_n^{m_1}}{2} & \dfrac{a_n^{m_1}-a_{3n}^{m_1}-b_n^{m_1}-b_{3n}^{m_1}}{2\sqrt{2}} & \dfrac{a_n^{m_1}+a_{3n}^{m_1}}{2} & \dfrac{a_n^{m_1}-a_{3n}^{m_1}+b_n^{m_1}+b_{3n}^{m_1}}{2\sqrt{2}} \\ \dfrac{a_n^{m_2}+a_{3n}^{m_2}}{2} & \dfrac{a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2}}{2\sqrt{2}} & \dfrac{b_n^{m_2}-b_{3n}^{m_2}}{2} & \dfrac{a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2}}{2\sqrt{2}} \\ \dfrac{b_{3n}^{m_3}-b_n^{m_3}}{2} & \dfrac{a_n^{m_3}-a_{3n}^{m_3}-b_n^{m_3}-b_{3n}^{m_3}}{2\sqrt{2}} & \dfrac{a_n^{m_3}+a_{3n}^{m_3}}{2} & \dfrac{a_n^{m_3}-a_{3n}^{m_3}+b_n^{m_3}+b_{3n}^{m_3}}{2\sqrt{2}} \end{vmatrix}=$$

$$\frac{1}{16}\begin{vmatrix} a_n^{m_1}-a_{3n}^{m_1}-b_n^{m_1}-b_{3n}^{m_1} & a_n^{m_1}+a_{3n}^{m_1} & a_n^{m_1}-a_{3n}^{m_1}+b_n^{m_1}+b_{3n}^{m_1} \\ a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2} & b_n^{m_2}-b_{3n}^{m_2} & a_n^{m_2}-a_{3n}^{m_2}+b_n^{m_2}+b_{3n}^{m_2} \\ a_n^{m_3}-a_{3n}^{m_3}-b_n^{m_3}-b_{3n}^{m_3} & a_n^{m_3}+a_{3n}^{m_3} & a_n^{m_3}-a_{3n}^{m_3}+b_n^{m_3}+b_{3n}^{m_3} \end{vmatrix}$$

利用Sharma-Varma算子的定义，再结合文献[8]中的方法讨论方程组的解，便可得到插值问题有解的充分必要条件，由插值条件容易得到 $T(x)$ 的如定理 1 所述的显式表达式.

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Solution on a Kind of Lacunary Interpolation by Antiperiodic Function

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Abstract：The 2-periodic interpolation under antiperiodic trigonometric polynomials is considered. A series of linear equations are established by decomposing the basis polynomials of interpolation，and a sufficient and necessary condition is given based on the Cramer's rule and the explicit expression of the interpolation solution is obtained.

Key words：antiperiodic function；2-periodic；interpolation

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