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# 基于犹豫度的优先加权平均算子在 区间犹豫模糊语言集下的多属性群决策

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**摘要:**针对评价信息为区间犹豫模糊语言集, 专家权重和属性权重完全未知的多属性群决策问题, 本文定义了区间犹豫模糊语言集上的一个新的得分函数, 提出了基于犹豫度的专家及属性优先关系的计算方法, 根据优先关系定义了优先加权平均(IVHFLHDPWA)算子, 由该算子得到综合评价信息计算得分值并进行排序, 最后将其应用于火灾应急预案的选择问题中, 验证了所提算子的有效性和适用性。

**关键词:** 区间犹豫模糊语言集; 犹豫度; IVHFLHDPWA 算子;  
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## Multi-Attribute Group Decision Making Based on Hesitant Degree Priority Weighted Average Operator under Interval Hesitation Fuzzy Linguistic Information

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**Abstract:** In this paper, the multi-attribute group decision-making problem on the interval hesitation fuzzy linguistic set is studied, in which the expert weights and attribute weights are completely unknown. First, we defined a new scoring function on the interval hesitation fuzzy linguistic set. Then, we proposed a method to calculate the priority relationship of experts and attributes. Finally, the priority weighted average based on hesitation degree on interval hesitation fuzzy linguistic set operator (IVHFLHDPWA) was

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proposed, and the score value of the comprehensive evaluation information was obtained. The validity and applicability of the proposed operator was verified by applying it to the multi-attribute group decision-making problem of fire emergency plan selection.

**Key words:** interval hesitant fuzzy linguistic set; hesitant degree; IVHFLHDPWA operator; multi-attribute group decision-making

多属性群决策<sup>[1]</sup>(MAGDM)作为决策领域的一个重要分支,是多个专家应用决策方法从含有多个属性的备选方案中择优的过程.在实际应用中,由于现实问题的不确定性以及决策者的偏好差异,方案的评价往往呈现多元化,使得最优方案的选取较为困难.因此,在多属性群决策问题中,先将多元化的评价信息进行集结,再选取最优方案的决策方法成为了研究者们研究热点.

文献[2]提出了模糊集(FS)的概念,其引入的隶属度在刻画事物确定度方面具有优越性,使得模糊集被广泛应用于不确定问题中.考虑到单一值刻画复杂问题的局限,文献[3]提出了犹豫模糊集(HFS)的概念,通过使用多个可能的隶属度来完整描述评价信息,随后犹豫模糊集上的运算<sup>[4]</sup>、距离测度<sup>[5]</sup>及相关系数的计算<sup>[6]</sup>等相继被研究.为了描述信息的不确定性,文献[7]提出了区间模糊集(IVFS)的概念,将隶属度拓展为区间形式,吸引了学者们的广泛研究<sup>[8-12]</sup>.文献[13]等结合犹豫模糊集和区间模糊集,定义了区间犹豫模糊集(IVHFS),其隶属度为所有可能的区间集合,能同时体现决策者的犹豫心理和事物的不确定性.文献[14-18]等研究了该集合的熵、距离和相似测度.

在决策问题中,有时候不需要对事物的描述进行量化,或者事物本身就很难量化,对于这类定性问题用语言术语<sup>[19]</sup>能更好进行表达.文献[20]提出了模糊语言集(FLS)的概念.为了体现决策者的犹豫性,用多个语言术语描述对象,文献[21]定义了犹豫模糊语言术语集(HFLTTS).后来,为了只用一个语言术语去描述对象,文献[22]提出了犹豫模糊语言集(HFLS),用犹豫模糊集描述该语言术语的准确程度.文献[23]考虑区间形式提出区间犹豫模糊语言集(IVHFLS)的概念,用一个犹豫的模糊区间集来描述语言术语的准确程度:一方面用语言术语描述对象,克服了难以量化的缺陷;另一方面用犹豫的区间描述语言术语的准确程度,同时考虑决策者的犹豫性和不确定性.关于 IVHFLS 的集结算子<sup>[24]</sup>、Shapley 模糊测度<sup>[25]</sup>、TOPSIS 方法<sup>[26]</sup>、语言尺度函数<sup>[27]</sup>等相继被研究.

上述这些研究中,属性权重或专家权重都是直接给出的,其结果具有较强的主观性.文献[23]研究了区间犹豫模糊语言集上的多属性决策问题,提出优先加权平均算子和优先加权几何算子,它的属性优先关系由决策者直接给出.本文研究区间犹豫模糊语言集上属性和专家的优先关系也完全未知的多属性群决策问题,考虑到犹豫度的大小可以反映专家对事物的认知程度,而专家的认知程度又影响专家及属性的优先关系,我们提出基于犹豫度来计算专家及属性的优先关系,保证更熟悉这一属性领域的专家和被认知程度更高的属性在决策中占有更大的权重.

## 1 预备知识

### 1.1 语言集与语言尺度函数

**定义 1**<sup>[19]</sup>(语言术语集) 语言术语集  $S = \{s_i \mid i = 0, 1, \dots, 2\tau\}$ , 满足如下运算:

1) 有序性: 如果  $i > j$ , 则  $s_i > s_j$ , 反之亦然;

2) 逆运算: 如果  $i + j = 2\tau$ , 则  $s_i = \text{neg}(s_j)$ .

其中语言术语的个数  $2\tau + 1$  称为该术语集的粒度,  $\text{neg}$  函数见定义 5.

**定义 2**<sup>[27]</sup> (语言尺度函数) 语言尺度函数  $f$  是语言术语到正实数上的一个映射, 定义一个连续语言下标到正实数上的语言尺度函数  $f: f(s_i) = \epsilon_i (i \in [0, 2\tau], \epsilon_i \in \mathbb{R}^+)$ , 这里的  $f$  为严格单调递增函数且存在反函数, 原函数及其反函数表达如下:

$$f(s_i) = \epsilon_i = \begin{cases} \frac{a^\tau - a^{\tau-i}}{2a^\tau - 2} & i \in [0, \tau] \\ \frac{a^\tau + a^{i-\tau} - 2}{2a^\tau - 2} & i \in (\tau, 2\tau] \end{cases}$$

$$f^{-1}(\epsilon_i) = \begin{cases} s_{\tau - \log_a(a^\tau - (2a^\tau - 2)\epsilon_i)} & \epsilon_i \in \left[0, \frac{1}{2}\right] \\ s_{\tau + \log_a((2a^\tau - 2)\epsilon_i - a^\tau + 2)} & \epsilon_i \in \left(\frac{1}{2}, 1\right] \end{cases} \quad (1)$$

函数  $f$  随着语言术语从中间扩展到两端, 相邻语言下标之间的绝对偏差增大 ( $a$  为由主观确定的值). 假设指标  $A$  比指标  $B$  重要得多, 且重要性比为  $m$ , 那么得到  $a = \sqrt[k]{m}$  (其中  $k$  表示语言集的粒度). 目前, 一般认为  $m$  上限为 9, 所以当语言集粒度为 7 时, 我们可以得到  $a = \sqrt[7]{9} \approx 1.37$ .

## 1.2 区间犹豫模糊语言集 (IVHFLS)

**定义 3**<sup>[23]</sup> 设  $X = \{x_1, x_2, \dots, x_n\}$  为给定对象集,  $s_{\theta(x)} \in S$  为描述对象  $x$  的语言术语, 则 IVHFLS 定义为

$$A = \{\langle x, s_{\theta(x)}, \Gamma_A(x) \rangle \mid x \in X\} \quad (2)$$

这里  $\Gamma_A(x)$  是  $(0, 1]$  上的有限子集的集合, 表示对象  $x$  满足语言术语  $s_{\theta(x)}$  的所有可能的模糊区间.

**定义 4**<sup>[5]</sup> 设  $A$  是  $X = \{x_1, x_2, \dots, x_n\}$  上的一个 IVHFLS,  $\alpha = \langle x_i, s_{\theta(x_i)}, \Gamma_A(x_i) \rangle$  是其中的一个元素, 称为区间犹豫模糊语言数 (IVHFLN), 为了方便书写, 我们也将 IVHFLN 记为  $\alpha = \langle s_{\theta(\alpha)}, \Gamma_\alpha \rangle$ .  $l(\alpha)$  表示  $\Gamma_A(x_i)$  中模糊区间的个数, 则 IVHFLN 和 IVHFLS 的犹豫度分别定义为:

$$\mu(\alpha) = 1 - \frac{1}{l(\alpha)} \quad \mu(A) = \frac{1}{n} \sum_{i=1}^n \mu(\alpha) \quad (3)$$

**定义 5**<sup>[23]</sup> 设  $\alpha = \langle s_{\theta(\alpha)}, \Gamma_\alpha \rangle$ ,  $\beta = \langle s_{\theta(\beta)}, \Gamma_\beta \rangle$  是两个 IVHFLN, 其基本运算如下:

$$1) \text{ neg}(\alpha) = \langle f^{-1}(f(s_{2\tau}) - f(s_{\theta(\alpha)})), \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \{[1 - r^l, 1 - r^u]\} \rangle;$$

$$2) \quad \alpha \oplus \beta = \langle f^{-1}(f(s_{\theta(\alpha)}) + f(s_{\theta(\beta)})), \bigcup_{r_1=[r_1^l, r_1^u] \in \Gamma_\alpha, r_2=[r_2^l, r_2^u] \in \Gamma_\beta} \left\{ \left[ \frac{f(s_{\theta(\alpha)})r_1^l + f(s_{\theta(\beta)})r_2^l}{f(s_{\theta(\alpha)}) + f(s_{\theta(\beta)})}, \frac{f(s_{\theta(\alpha)})r_1^u + f(s_{\theta(\beta)})r_2^u}{f(s_{\theta(\alpha)}) + f(s_{\theta(\beta)})} \right] \right\} \rangle;$$

$$3) \lambda\alpha = \langle f^{-1}(\lambda f(s_{\theta(\alpha)})), \Gamma_\alpha \rangle;$$

$$4) \alpha \otimes \beta = \langle f^{-1}(f(s_{\theta(\alpha)})f(s_{\theta(\beta)})), \bigcup_{r_1=[r_1^l, r_1^u] \in \Gamma_\alpha, r_2=[r_2^l, r_2^u] \in \Gamma_\beta} \{[r_1^l r_2^l, r_1^u r_2^u]\} \rangle;$$

$$5) \alpha^\lambda = \langle f^{-1}((f(s_{\theta(\alpha)}))^\lambda), \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \{[(r^l)^\lambda, (r^u)^\lambda]\} \rangle.$$

**定义 6**<sup>[23]</sup> 设  $\alpha = \langle s_{\theta(\alpha)}, \Gamma_\alpha \rangle = \langle s_{\theta(\alpha)}, \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \{[r^l, r^u]\} \rangle$  是一个 IVHFLN,  $E_r(\Gamma_\alpha)$  为  $\Gamma_\alpha$  的期望

函数,  $E_r(\Gamma_\alpha) = \frac{\sum_{[r^l, r^u] \in \Gamma_\alpha} (r^l + r^u)}{2l(\alpha)}$ ,  $D_r(\Gamma_\alpha)$  为  $\Gamma_\alpha$  的方差函数,

$$D_r(\Gamma_\alpha) = \frac{1}{l(\alpha)} \sum_{[r^l, r^u] \in \Gamma_\alpha} \left( \frac{r^l + r^u}{2} - E_r(\Gamma_\alpha) \right)^2$$

则  $\alpha$  的得分函数和精确函数分别定义为

$$E(\alpha) = f(s_{\theta(\alpha)}) \times E_r(\Gamma_\alpha) \quad (4)$$

$$D(\alpha) = f(s_{\theta(\alpha)}) \times (1 - D_r(\Gamma_\alpha)) \quad (5)$$

$l(\alpha)$  表示  $\Gamma_\alpha$  中子集的个数.

设  $\alpha = \langle s_{\theta(\alpha)}, \Gamma_\alpha \rangle$  和  $\beta = \langle s_{\theta(\beta)}, \Gamma_\beta \rangle$  是两个 IVHFLN, 其比较规则如下:

- 1) 如果  $E(\alpha) > E(\beta)$ , 则  $\alpha > \beta$ ;
- 2) 如果  $E(\alpha) = E(\beta)$  且  $D(\alpha) > D(\beta)$ , 则  $\alpha > \beta$ ;
- 3) 如果  $E(\alpha) = E(\beta)$  且  $D(\alpha) = D(\beta)$ , 则  $\alpha = \beta$ .

## 2 基于犹豫度的优先加权平均算子

IVHFLN 的比较是一个重要问题, 文献[23] 中区间犹豫模糊语言数的比较规则考虑了语言术语的大小和犹豫区间的均值, 我们考虑到犹豫度对得分函数的影响定义了新的得分函数, 同时考虑了信息的犹豫度、区间期望和语言术语的大小, 得到综合得分函数, 给出如下定义.

**定义 7** 设  $\alpha = \langle s_{\theta(\alpha)}, \Gamma_\alpha \rangle = \langle s_{\theta(\alpha)}, \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \{[r^l, r^u]\} \rangle$  是一个 IVHFLN, 犹豫区间的期望函数为

$$IE(\Gamma_\alpha) = \frac{\sum_{[r^l, r^u] \in \Gamma_\alpha} (r^l + r^u)}{2l(\alpha)}$$

则考虑犹豫度的新的得分函数定义如下:

$$S(\alpha) = e^{\frac{1}{l(\alpha)} - 1} \times f(s_{\theta(\alpha)}) \times IE(\Gamma_\alpha) \quad (6)$$

其中  $l(\alpha)$  表示  $\Gamma_\alpha$  中子集的个数.

### 2.1 基于犹豫度的优先加权平均算子

文献[28] 提出优先集结算子, 它的核心思想是用优先关系来计算权重, 其属性的优先关系由决策者直接给出, 本文研究优先关系也未知的问题, 根据犹豫度计算优先关系, 得到基于犹豫度的优先加权平均算子, 并将其应用于区间犹豫模糊语言集, 定义区间模糊犹豫语言集基于犹豫度的优先加权平均算子, 表达形式如下:

**定义 8** 对  $n$  个区间犹豫模糊语言数, 根据犹豫度将其划分为具有优先关系的  $q$  类  $G_i(x) (i=1, 2, \dots, q)$ , 且满足  $G_1(x) \succ G_2(x) \succ \dots \succ G_q(x)$ , 即当  $j < k$  时  $G_k(x)$  优先于  $G_j(x)$  考虑. 令  $G_i(x) = \{C_{i1}(x), C_{i2}(x), \dots, C_{in_i}(x)\}$ ,  $n = \sum_{i=1}^q n_i$ , 则区间犹豫模糊语言集基于犹豫度的优先加权平均(IVHFLHDPWA)算子定义为

$$\text{IVHFLHDPWA}(G_1(x), G_2(x), \dots, G_q(x)) = \bigoplus_{i=1}^q \bigoplus_{j=1}^{n_i} \omega_i C_{ij}(x) \quad (7)$$

其中  $\omega_i$  表示类别  $G_i(x)$  的权重, 由优先关系计算得到,  $\omega_i = \frac{T_i}{\sum_{i=1}^q T_i}$ ,  $(i=1, 2, \dots, q)$ ,  $\sum_{i=1}^q \omega_i = 1$ ,  $T_1 = 1$ ,

$T_i = T_{i-1} \max\{E(C_{i-1,1}(x)), E(C_{i-1,2}(x)), \dots, E(C_{i-1,n_{i-1}}(x))\} (i=2, 3, \dots, q)$ ,  $E(C_{i1}(x))$  表示  $C_{i1}(x)$  的得分函数.

### 2.2 基于犹豫度的优先加权平均算子的性质

**定理 1** 设有优先关系的  $q$  个集合  $G_i(x) (i=1, 2, \dots, q)$  为一个 IVHFLS,  $G_i(x) = \{C_{i1}(x), C_{i2}(x), \dots,$

$C_{in_i}(x)\}$ ,  $n = \sum_{i=1}^q n_i$ ; 记  $C_{ij}(x) = \langle s_{\theta(c_{ij})}, \Gamma_{c_{ij}} \rangle$ , 则 IVHFLHDPWA 算子集结结果仍然为一个 IVHFLN, 且有如下形式:

$$\text{IVHFLHDPWA}(G_1, G_2, \dots, G_q) = \left\langle f^{-1} \left( \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right) \right), \right. \\ \left. \bigcup_{i=1}^q r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \right. \\ \left. \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle \quad (8)$$

证 根据定义 5 中区间犹豫模糊语言数的运算得:

$$\omega_i C_{ij}(x) = \langle f^{-1}(\omega_i f(s_{\theta(c_{ij})})), \Gamma_{c_{ij}} \rangle$$

从而

$$\bigoplus_{j=1}^{n_i} \omega_i C_{ij}(x) = \left\langle f^{-1} \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right), \bigcup_{i=1}^q r_{i1} = [r_{i1}^l, r_{i1}^u] \in \Gamma_{c_i}, \dots, r_{in_i} = [r_{in_i}^l, r_{in_i}^u] \in \Gamma_{c_i} \right\{ \left[ \frac{\sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle$$

所以

$$\bigoplus_{i=1}^q \bigoplus_{j=1}^{n_i} \omega_i C_{ij}(x) = \left\langle f^{-1} \left( \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right) \right), \right. \\ \left. \bigcup_{i=1}^q r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \right. \\ \left. \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle$$

**定理 2** 对  $n$  个 IVHFLN, 基于犹豫度将其划分为具有优先关系的  $q$  类  $G_i(x) (i=1, 2, \dots, q)$ , 且满足  $G_1(x) \succ G_2(x) \succ \dots \succ G_q(x)$ , 令

$$G_i(x) = \{C_{i1}(x), C_{i2}(x), \dots, C_{in_i}(x)\}, n = \sum_{i=1}^q n_i$$

我们记

$$C_{ij}(x) = \langle s_{\theta(c_{ij})}, \Gamma_{c_{ij}} \rangle$$

则对任意的  $r > 0$ , 有

$$\text{IVHFLHDPWA}(rG_1(x), rG_2(x), \dots, rG_q(x)) = r\text{IVHFLHDPWA}(G_1(x), G_2(x), \dots, G_q(x)) \quad (9)$$

证  $C_{ij}(x) = \langle f^{-1}(rf(s_{\theta(c_{ij})})), \Gamma_{c_{ij}} \rangle$ , 一方面, 根据公式(8)

$$\begin{aligned}
& \text{IVHFLHDPWA}(rG_1(x), rG_2(x), \dots, rG_q(x)) = \\
& \left( \bigoplus_{i=1}^q \bigoplus_{j=1}^{n_i} \omega_i rC_{ij}(x) = \left\langle f^{-1} \left( \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i r f(s_{\theta(c_{ij})}) \right) \right) \right\rangle, \right. \\
& r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \\
& \left. \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle \\
& = \left\langle f^{-1} \left( r \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right) \right) \right\rangle, \\
& r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \\
& \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle
\end{aligned}$$

另一方面

$$\begin{aligned}
& r\text{IVHFLHDPWA}(G_1(x), G_2(x), \dots, G_q(x)) = \\
& r \left\langle f^{-1} \left( \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right) \right) \right\rangle, \\
& r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \\
& \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle \\
& = \left\langle f^{-1} \left( r \sum_{i=1}^q \left( \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) \right) \right) \right\rangle, \\
& r_{11} = [r_{11}^l, r_{11}^u] \in \Gamma_{c_1}, \dots, r_{1n_1} = [r_{1n_1}^l, r_{1n_1}^u] \in \Gamma_{c_1}, \dots, r_{q1} = [r_{q1}^l, r_{q1}^u] \in \Gamma_{c_q}, \dots, r_{qn_q} = [r_{qn_q}^l, r_{qn_q}^u] \in \Gamma_{c_{qr}} \\
& \left\{ \left[ \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^l}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})}, \frac{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})}) r_{ij}^u}{\sum_{i=1}^q \sum_{j=1}^{n_i} \omega_i f(s_{\theta(c_{ij})})} \right] \right\} \right\rangle
\end{aligned}$$

**性质 1(幂等性)** 对  $n$  个区间犹豫模糊语言数  $\alpha_i (i = 1, 2, \dots, n)$ , 如果对所有  $i$  都有  $\alpha_i = \alpha$ , 则

$$\text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \alpha.$$

**证** 因为每个区间犹豫模糊语言数相等, 则它们的优先关系一样, 由定义 8 权重计算方法可知其权重相等, 再由定义 5 中的运算得到:

$$\begin{aligned} \text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) &= \text{IVHFLHDPWA}(\alpha, \alpha, \dots, \alpha) = \underbrace{\frac{1}{n}\alpha \oplus \dots \oplus \frac{1}{n}\alpha}_n = \\ &= \left\langle f^{-1}\left(\frac{1}{n}f(s_{\theta(\alpha)})\right), \Gamma_\alpha \right\rangle \oplus \dots \oplus \left\langle f^{-1}\left(\frac{1}{n}f(s_{\theta(\alpha)})\right), \Gamma_\alpha \right\rangle = \\ &= \left\langle f^{-1}\left(\frac{1}{n}f(s_{\theta(\alpha)}) + \dots + \frac{1}{n}f(s_{\theta(\alpha)})\right) \right\rangle, \\ &= \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \left\langle \left[ \frac{\frac{1}{n}f(s_{\theta(\alpha)})r^l + \frac{1}{n}f(s_{\theta(\alpha)})r^l}{\frac{1}{n}f(s_{\theta(\alpha)}) + \frac{1}{n}f(s_{\theta(\alpha)})}, \frac{\frac{1}{n}f(s_{\theta(\alpha)})r^u + \frac{1}{n}f(s_{\theta(\alpha)})r^u}{\frac{1}{n}f(s_{\theta(\alpha)}) + \frac{1}{n}f(s_{\theta(\alpha)})} \right] \right\rangle = \\ &= \langle s_{\theta(\alpha)}, \bigcup_{r=[r^l, r^u] \in \Gamma_\alpha} \{[r^l, r^u]\} \rangle = \alpha \end{aligned}$$

**性质 2(交换性)** 设有  $n$  个区间犹豫模糊语言数  $\alpha_i (i=1, 2, \dots, n)$ ,  $(\beta_1, \beta_2, \dots, \beta_n)$  是  $(\alpha_1, \alpha_2, \dots, \alpha_n)$  的任意一个置换, 则有  $\text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \text{IVHFLHDPWA}(\beta_1, \beta_2, \dots, \beta_n)$ .

**证** 对  $\alpha_i$  的任意一个置换, 集结时先根据犹豫度计算出优先关系, 处于同一优先级则有相同的权重, 因而集结结果与区间犹豫模糊语言数的排列顺序无关.

**性质 3(有界性)** 对一个区间犹豫模糊语言集  $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ , 基于犹豫度划分为具有优先关系的  $q$  类,  $G_i(x) (i=1, 2, \dots, q)$ , 其中  $G_i(x) = \{C_{i1}(x), C_{i2}(x), \dots, C_{in_i}(x)\}$ ,  $n = \sum_{i=1}^q n_i$ . 令  $\alpha_m = \min_i \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ ,  $\alpha_M = \max_i \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ , 则有  $\alpha_m \leq \text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) \leq \alpha_M$ .

**证** 由性质 1 可以得到:

$$\begin{aligned} &\text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \\ &\text{IVHFLHDPWA}(G_1(x), G_2(x), \dots, G_q(x)) = \\ &= \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{j=1}^{n_i} C_{ij}(x) \right) \geq \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{j=1}^{n_i} \alpha_m \right) = \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \alpha_m \right) = \\ &= \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{i=1}^q \alpha_m = \alpha_m \\ &\text{IVHFLHDPWA}(\alpha_1, \alpha_2, \dots, \alpha_n) = \\ &\text{IVHFLHDPWA}(G_1(x), G_2(x), \dots, G_q(x)) = \\ &= \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{j=1}^{n_i} C_{ij}(x) \right) \leq \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{j=1}^{n_i} \alpha_M \right) = \\ &= \bigoplus_{i=1}^q \left( \frac{T_i}{\sum_{i=1}^q T_i} \alpha_M \right) = \frac{T_i}{\sum_{i=1}^q T_i} \bigoplus_{i=1}^q \alpha_M = \alpha_M \end{aligned}$$

### 3 基于犹豫度的优先加权平均算子的多属性群决策

对一个多属性群决策问题, 有  $m$  个方案  $A = \{A_1, A_2, \dots, A_m\}$ 、 $n$  个属性  $C = \{C_1, C_2, \dots, C_n\}$  和  $s$  个专家  $D = \{D_1, D_2, \dots, D_s\}$ , 每个专家  $D_k$  根据自己的专业知识和偏好关系对每个方案  $A_i$  的每个属性  $C_j$  进行评价, 评价结果用区间犹豫模糊语言数表示, 则基于新的得分函数的区间犹豫模糊语言集多属性群决策基本步骤如下:

1) 建立区间犹豫模糊语言群决策矩阵并将其标准化.

用一个 IVHFLN 表示专家  $D_k$  对方案  $A_i$  的属性  $C_j$  的评价值并记为  $\tilde{h}_{ij}^k$ , 这里  $\tilde{h}_{ij}^k = \langle \tilde{s}_{\theta(h_{ij}^k)}, \tilde{\Gamma}_{h_{ij}^k} \rangle$ , 从而  $s$  个专家对  $m$  个方案的  $n$  个属性的评价结果用决策矩阵表示为:

$$\tilde{\mathbf{H}}^k = \begin{bmatrix} \tilde{h}_{11}^k & \tilde{h}_{12}^k & \cdots & \tilde{h}_{1n}^k \\ \tilde{h}_{21}^k & \tilde{h}_{22}^k & \cdots & \tilde{h}_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{h}_{m1}^k & \tilde{h}_{m2}^k & \cdots & \tilde{h}_{mn}^k \end{bmatrix} \quad k = 1, 2, \dots, s$$

由于属性可以分为效益型和成本型, 为了便于计算, 首先对决策矩阵标准化. 对成本型属性  $h_{ij}^k = \text{neg}(\tilde{h}_{ij}^k)$ , 对效益型属性  $h_{ij}^k = \tilde{h}_{ij}^k$ , 得到标准化群决策矩阵  $\tilde{\mathbf{H}}^k = (h_{ij}^k)_{m \times n}$  ( $k = 1, 2, \dots, s$ ).

2) 根据犹豫度计算专家集优先关系.

与传统的给定专家优先关系不同, 根据专家对方案属性评价值的犹豫度来得到属性下专家的优先关系, 犹豫度越小说明专家在该属性下的认知程度越高, 优先级也就越高, 每个专家在各属性下的犹豫度由公式(3)得到:

$$\mu(H_j^k) = \frac{1}{m} \sum_{i=1}^m \mu(h_{ij}^k) \quad \mu(h_{ij}^k) = 1 - \frac{1}{l(h_{ij}^k)} \quad (10)$$

其中:  $l(h_{ij}^k)$  表示  $\Gamma_{h_{ij}^k}$  中子集的个数;  $k = 1, 2, \dots, s$ ;  $j = 1, 2, \dots, n$ .

3) 集结群决策矩阵.

运用 IVHFLHDPWA 算子对多个专家的评价信息进行集结, 得到一个综合决策矩阵  $\mathbf{H} = (h_{ij})_{m \times n}$ :

$$\text{IVHFLHDPWA}(h_{ij}^1, h_{ij}^2, \dots, h_{ij}^s) = h_{ij} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (11)$$

4) 根据公式(3)计算犹豫度, 进而得到综合决策矩阵在每个方案下的属性优先关系.

$$\mu(H_j) = \frac{1}{m} \sum_{i=1}^m \mu(h_{ij}) \quad j = 1, 2, \dots, n$$

$$\mu(h_{ij}) = 1 - \frac{1}{l(h_{ij})} \quad (12)$$

其中  $l(h_{ij})$  表示  $\Gamma_{h_{ij}}$  中子集的个数.

5) 集结方案的属性值.

在 4) 的基础上, 基于属性优先关系运用 IVHFLHDPWA 算子对综合决策矩阵每个方案的属性值进行集结, 得到方案的综合评价, 即:

$$\text{IVHFLHDPWA}(h_{i1}, h_{i2}, \dots, h_{in}) = h_i \quad i = 1, 2, \dots, m \quad (13)$$

6) 根据公式(6)计算方案  $A_i$  的得分值  $S(h_i)$ .

7) 由得分值从大到小对方案进行排序.



4 实例分析

火灾应急方案的选择是处理突发火灾事故的一项重要工作,能够有效保障生命财产安全.假设某公司有 5 个待决策的火灾应急方案 $\{x_1, x_2, x_3, x_4, x_5\}$ ,一个有 3 名专家的决策团队对这 5 种应急方案进行评估,选出最好的应急方案.考虑火灾应急的 4 个主要属性: $c_1$  为预测能力; $c_2$  为救援能力; $c_3$  为响应速度; $c_4$  为指挥能力.每个专家对每种方案的每一个主要属性进行评价,其评价值用区间犹豫模糊语言数表示,其中语言术语采用 7 值语言集  $S = \{s_0, s_1, \cdots, s_6\}$  来刻画<sup>[29]</sup>.其具体步骤如下:

1) 3 位专家分别对 5 种火灾应急方案基于 4 个属性指标进行评价,评价结果见表 1.

表 1 专家决策矩阵

| 方案    | $c_1$                                             | $c_2$                                             | $c_3$                                             | $c_4$                                             |
|-------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| $x_1$ | $\langle s_4, \{[0.5, 0.7]\} \rangle$             | $\langle s_4, \{[0.3, 0.4], [0.5, 0.6]\} \rangle$ | $\langle s_3, \{[0.3, 0.4], [0.5, 0.7]\} \rangle$ | $\langle s_4, \{[0.4, 0.6]\} \rangle$             |
| $x_2$ | $\langle s_2, \{[0.4, 0.5], [0.5, 0.6]\} \rangle$ | $\langle s_3, \{[0.4, 0.5]\} \rangle$             | $\langle s_2, \{[0.3, 0.6]\} \rangle$             | $\langle s_4, \{[0.3, 0.4], [0.5, 0.6]\} \rangle$ |
| $x_3$ | $\langle s_4, \{[0.3, 0.5], [0.7, 0.8]\} \rangle$ | $\langle s_5, \{[0.4, 0.5]\} \rangle$             | $\langle s_4, \{[0.6, 0.8]\} \rangle$             | $\langle s_4, \{[0.5, 0.6]\} \rangle$             |
| $x_4$ | $\langle s_2, \{[0.2, 0.4], [0.5, 0.7]\} \rangle$ | $\langle s_3, \{[0.3, 0.4], [0.6, 0.7]\} \rangle$ | $\langle s_3, \{[0.6, 0.7]\} \rangle$             | $\langle s_4, \{[0.4, 0.5]\} \rangle$             |
| $x_5$ | $\langle s_5, \{[0.5, 0.7]\} \rangle$             | $\langle s_5, \{[0.4, 0.6]\} \rangle$             | $\langle s_4, \{[0.4, 0.5], [0.7, 0.8]\} \rangle$ | $\langle s_5, \{[0.6, 0.7]\} \rangle$             |

| 方案    | $c_1$                                             | $c_2$                                             | $c_3$                                             | $c_4$                                             |
|-------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| $x_1$ | $\langle s_2, \{[0.4, 0.6]\} \rangle$             | $\langle s_2, \{[0.5, 0.6], [0.6, 0.7]\} \rangle$ | $\langle s_3, \{[0.6, 0.7], [0.7, 0.8]\} \rangle$ | $\langle s_3, \{[0.3, 0.6]\} \rangle$             |
| $x_2$ | $\langle s_3, \{[0.4, 0.5], [0.6, 0.8]\} \rangle$ | $\langle s_3, \{[0.6, 0.8]\} \rangle$             | $\langle s_3, \{[0.5, 0.7]\} \rangle$             | $\langle s_4, \{[0.5, 0.6]\} \rangle$             |
| $x_3$ | $\langle s_5, \{[0.4, 0.6], [0.7, 0.8]\} \rangle$ | $\langle s_3, \{[0.5, 0.6]\} \rangle$             | $\langle s_4, \{[0.4, 0.5]\} \rangle$             | $\langle s_3, \{[0.4, 0.5], [0.6, 0.7]\} \rangle$ |
| $x_4$ | $\langle s_5, \{[0.6, 0.7]\} \rangle$             | $\langle s_5, \{[0.4, 0.5], [0.5, 0.6]\} \rangle$ | $\langle s_5, \{[0.7, 0.8]\} \rangle$             | $\langle s_4, \{[0.6, 0.8]\} \rangle$             |
| $x_5$ | $\langle s_4, \{[0.5, 0.6]\} \rangle$             | $\langle s_4, \{[0.3, 0.4], [0.6, 0.7]\} \rangle$ | $\langle s_5, \{[0.4, 0.7]\} \rangle$             | $\langle s_3, \{[0.6, 0.7]\} \rangle$             |

| 方案    | $c_1$                                             | $c_2$                                             | $c_3$                                             | $c_4$                                             |
|-------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| $x_1$ | $\langle s_2, \{[0.6, 0.7]\} \rangle$             | $\langle s_4, \{[0.6, 0.8]\} \rangle$             | $\langle s_3, \{[0.5, 0.6]\} \rangle$             | $\langle s_3, \{[0.3, 0.4], [0.6, 0.7]\} \rangle$ |
| $x_2$ | $\langle s_4, \{[0.5, 0.6]\} \rangle$             | $\langle s_4, \{[0.4, 0.5], [0.6, 0.8]\} \rangle$ | $\langle s_4, \{[0.6, 0.7]\} \rangle$             | $\langle s_3, \{[0.2, 0.4], [0.4, 0.6]\} \rangle$ |
| $x_3$ | $\langle s_2, \{[0.4, 0.6]\} \rangle$             | $\langle s_3, \{[0.7, 0.8]\} \rangle$             | $\langle s_2, \{[0.4, 0.6]\} \rangle$             | $\langle s_2, \{[0.4, 0.7]\} \rangle$             |
| $x_4$ | $\langle s_4, \{[0.5, 0.7]\} \rangle$             | $\langle s_5, \{[0.5, 0.6], [0.6, 0.8]\} \rangle$ | $\langle s_4, \{[0.2, 0.4], [0.6, 0.8]\} \rangle$ | $\langle s_4, \{[0.3, 0.6]\} \rangle$             |
| $x_5$ | $\langle s_5, \{[0.3, 0.5], [0.5, 0.7]\} \rangle$ | $\langle s_4, \{[0.7, 0.8]\} \rangle$             | $\langle s_4, \{[0.4, 0.6], [0.6, 0.7]\} \rangle$ | $\langle s_3, \{[0.5, 0.7]\} \rangle$             |

因为 4 个属性指标均为效益属性,所以标准化结果不变.

2) 对于得到的群决策矩阵,由公式(3) 计算专家在每个属性下的犹豫度,从而得到专家优先关系见表 2.

表 2 各属性下专家的优先关系

| 属性   | 优先顺序                |
|------|---------------------|
| 预测能力 | 专家三优先于专家二,专家二优先于专家一 |
| 救援能力 | 专家一等同于专家三,专家三优先于专家二 |
| 响应速度 | 专家二优先于专家三,专家三等同于专家一 |
| 指挥能力 | 专家一等同于专家二,专家二优先于专家三 |

3) 基于专家优先关系用 IVHFLHDPWA 算子集结各专家决策矩阵得到综合决策矩阵(表 3).

表 3 专家综合决策矩阵

| 方案    | $c_1$                                                                                  | $c_2$                                                                                                                                          |
|-------|----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| $x_1$ | $\langle s_{2.06}, \{[0.56, 0.68]\} \rangle$                                           | $\langle s_{3.68}, \{[0.46, 0.60], [0.47, 0.61], [0.54, 0.69], [0.56, 0.70]\} \rangle$                                                         |
| $x_2$ | $\langle s_{3.71}, \{[0.48, 0.58], [0.52, 0.64], [0.48, 0.58], [0.52, 0.64]\} \rangle$ | $\langle s_{3.49}, \{[0.42, 0.53], [0.52, 0.68]\} \rangle$                                                                                     |
| $x_3$ | $\langle s_{2.57}, \{[0.39, 0.59], [0.47, 0.65], [0.42, 0.61], [0.50, 0.66]\} \rangle$ | $\langle s_{4.00}, \{[0.52, 0.62]\} \rangle$                                                                                                   |
| $x_4$ | $\langle s_{4.09}, \{[0.48, 0.68], [0.50, 0.70]\} \rangle$                             | $\langle s_{4.28}, \{[0.42, 0.52], [0.47, 0.62], [0.43, 0.53], [0.49, 0.64], [0.52, 0.62], [0.57, 0.72], [0.53, 0.63], [0.58, 0.74]\} \rangle$ |
| $x_5$ | $\langle s_{4.84}, \{[0.34, 0.53], [0.50, 0.69]\} \rangle$                             | $\langle s_{4.44}, \{[0.49, 0.64], [0.54, 0.69]\} \rangle$                                                                                     |

| 方案    | $c_3$                                                                                  | $c_4$                                                                                  |
|-------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| $x_1$ | $\langle s_{3.00}, \{[0.54, 0.64], [0.61, 0.71], [0.57, 0.69], [0.64, 0.76]\} \rangle$ | $\langle s_{3.00}, \{[0.35, 0.57], [0.39, 0.61]\} \rangle$                             |
| $x_2$ | $\langle s_{3.00}, \{[0.49, 0.69]\} \rangle$                                           | $\langle s_{3.87}, \{[0.38, 0.49], [0.40, 0.51], [0.46, 0.58], [0.49, 0.60]\} \rangle$ |
| $x_3$ | $\langle s_{3.64}, \{[0.51, 0.65]\} \rangle$                                           | $\langle s_{3.32}, \{[0.45, 0.57], [0.53, 0.65]\} \rangle$                             |
| $x_4$ | $\langle s_{4.28}, \{[0.55, 0.68], [0.65, 0.78]\} \rangle$                             | $\langle s_{4.00}, \{[0.46, 0.64]\} \rangle$                                           |
| $x_5$ | $\langle s_{4.57}, \{[0.40, 0.64], [0.44, 0.66], [0.46, 0.70], [0.50, 0.72]\} \rangle$ | $\langle s_{3.95}, \{[0.58, 0.70]\} \rangle$                                           |

4) 根据犹豫度计算综合决策矩阵在每个方案下的属性优先关系,得到的综合决策矩阵属性优先关系从高到低为:指挥能力、响应速度、预测能力、救援能力.

5) 基于属性优先关系运用 IVHFLHDPWA 算子对综合决策矩阵每个方案的属性值进行集结,得到方案的综合评价价值(表 4).

表 4 综合评价价值

| 方案    | 综合评价价值                                                                                                                                                 |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| $x_1$ | $\langle s_{2.98}, \{[0.38, 0.58]4, [0.38, 0.59]4, [0.39, 0.59]4, [0.39, 0.60]4, [0.41, 0.62]4, [0.42, 0.62]4, [0.42, 0.63]4, [0.43, 0.63]4\} \rangle$ |
| $x_2$ | $\langle s_{3.77}, \{[0.39, 0.51]8, [0.41, 0.53]8, [0.47, 0.59]8, [0.49, 0.61]8\} \rangle$                                                             |
| $x_3$ | $\langle s_{3.35}, \{[0.46, 0.58]2, [0.46, 0.59]2, [0.52, 0.65]3, [0.53, 0.65]1\} \rangle$                                                             |
| $x_4$ | $\langle s_{4.04}, \{[0.49, 0.65]14, [0.49, 0.66]2, [0.51, 0.68]16\} \rangle$                                                                          |
| $x_5$ | $\langle s_{4.06}, \{[0.52, 0.67]2, [0.52, 0.68]1, [0.53, 0.68]3, [0.53, 0.69]2, [0.54, 0.69]2, [0.54, 0.70]2, [0.55, 0.70]3, [0.56, 0.70]1\} \rangle$ |

这里,区间数后面的数字表示犹豫集中该区间数的个数.

6) 计算每个方案的得分值(表 5).

表 5 方案得分值

| 方案    | 方案得分值    | 方案    | 方案得分值    |
|-------|----------|-------|----------|
| $x_1$ | 0.095 71 | $x_4$ | 0.137 93 |
| $x_2$ | 0.111 60 | $x_5$ | 0.144 90 |
| $x_3$ | 0.123 95 |       |          |

7) 由得分值得到方案排序从高到低依次为  $x_5, x_4, x_3, x_2, x_1$ , 最优方案为  $x_5$ .

5 结束语

本文提出区间犹豫模糊语言集基于犹豫度的优先加权平均算子来处理专家和属性权重完全未知的多属

性群决策问题,让专家在自己熟悉的领域更有决策权.选取区间犹豫模糊语言集描述决策者对方案的评价,可以同时体现决策者的犹豫心理和对象的不确定性.与已有的决策方法相比,用定义的基于犹豫度的优先加权平均算子作决策,不仅考虑了犹豫度大小对决策的影响使得结果更加符合决策者心理,而且计算过程简单.本文所提出的方法也可以运用到工程选址、企业风险投资和绩效评定等其他决策问题中.

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