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不同观测点重尾参数的估计①

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摘要：根据位置不变的 Hill 型估计量的渐近性质，提出了一个关于极端降雨不同观测点的位置不变的估计量 $\hat{c}_n(k_0, k)$，并讨论了其弱相合性及其分布的渐近正态展开.

关 键 词：重尾分布；极值指数；位置不变；Hill 估计

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设 $X_1(s_j), X_2(s_j), \cdots, X_n(s_j)$ 是来自离散时间点 $0 = s_0 < s_1 < \cdots < s_m$ 的样本观测值，并且 $X_1(s_j)$，$X_2(s_j), \cdots, X_n(s_j)$ 为独立同分布随机变量序列，公共分布函数为 $F_{s_j}(x)$. $X_{1,n}(s_j) \leqslant X_{2,n}(s_j) \leqslant \cdots \leqslant X_{n,n}(s_j)$ 为 $X_1(s_j), X_2(s_j), \cdots, X_n(s_j)$ 的顺序统计量. 假设 $1 - F_0 \in RV_{-\frac{1}{\gamma}}$，$\gamma > 0$，

$$\lim_{x\to\infty} \frac{1 - F_{s_j}(x)}{1 - F_0(x)} = e^{cs_j}, \ j = 1,2,\cdots,m \tag{1}$$

对极值指数 γ 和参数 c 的估计有其理论及应用价值. 对 γ 的估计有 Hill 型估计及文献[1]提出的位置不变的 Hill 型估计：

$$\gamma_n^H(k_0, k) = \frac{1}{k_0} \sum_{i=0}^{k_0-1} \log \frac{X_{n-i,n}(0) - X_{n-k,n}(0)}{X_{n-k_0,n}(0) - \log X_{n-k,n}(0)} \tag{2}$$

对参数 c，文献[2]得到了如下的最小二乘估计量：

$$\hat{c} := \frac{\sum_{j=1}^{m} s_j (\log X_{n-k,n}(s_j) - \log X_{n-k,n}(0))}{\hat{\gamma}_{n,k} \sum_{j=1}^{m} s_j^2} \tag{3}$$

并应用于极端降雨模型，其中 $\hat{\gamma}_{n,k}$ 为 γ 的 Hill 型估计量.

基于文献[1]和文献[2]的工作，本文提出参数(γ, c)的位置不变估计量$(\gamma_n^H(k_0, k), \hat{c}_n(k_0, k))$并研究其渐近性质. 其中 $\gamma_n^H(k_0, k)$ 由(2)式给出，$\hat{c}_n(k_0, k)$ 定义如下：

$$\hat{c}_n(k_0, k) := \frac{1}{\hat{\gamma}_n(k_0, k) \sum_{j=1}^{m} s_j^2} \sum_{j=1}^{m} s_j \log\left(\frac{X_{n-k_0,n}(s_j) - X_{n-k,n}(s_j)}{X_{n-k_0,n}(0) - X_{n-k,n}(0)} \right) \tag{4}$$

当 $n \to \infty$ 时，k, k_0 满足

$$k_0 = k_0(k) \to \infty, \ k = k(n) \to \infty, \ \frac{k_0}{k} \to 0, \ \frac{k}{n} \to 0 \tag{5}$$

定理 1 假设 $1 - F \in RV_{-\frac{1}{\gamma}}$，$\gamma > 0$ 及(1)式成立. 则在(5)式的条件下，$\hat{c}_n(k_0, k) \xrightarrow{P} c$.

证 设 $Y_1, \cdots, Y_n$ 为服从标准帕累托分布 $F_Y(y) = 1 - \frac{1}{y} (y \geqslant 1)$ 的独立同分布的随机变量，$Y_{1,n} \leqslant$

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$Y_{2,n} \leqslant \cdots \leqslant Y_{n,n}$ 为其顺序统计量，$U_{s_j}(t) := \left(\frac{1}{1-F_{s_j}}\right)^{\leftarrow}$，则

$$\left\{\frac{Y_{n-i,n}}{Y_{n-k,n}}\right\}_{i=0}^{k-1} \overset{d}{=} \{Y_{k-i,k}\}_{i=0}^{k-1} \tag{6}$$

$$\{X_{n-i,n}\}_{i=0}^{k-1} \overset{d}{=} \{U(Y_{k-i,k})\}_{i=0}^{k-1} \tag{7}$$

(证明参见文献[5]推论 2.2.2).

由(3)式可得

$$\hat{C}_n(k_0, k) \overset{d}{=} \frac{1}{\hat{\gamma}_n(k_0, k)\sum_{j=1}^{m} s_j^2}\sum_{j=1}^{m} s_j \log\left(\frac{U_{s_j}(Y_{n-k_0,n}) - U_{s_j}(Y_{n-k,n})}{U_0(Y_{n-k_0,n}) - U_0(Y_{n-k,n})}\right) =$$

$$\frac{1}{\hat{\gamma}_n(k_0, k)\sum\limits_{j=1}^{m} s_j^2}\sum_{j=1}^{m} s_j \log\left(\frac{e^{c\gamma S_j}(1+o_p(1))(U_0(Y_{n-k_0,n}) - U_0(Y_{n-k,n}))}{U_0(Y_{n-k_0,n}) - U_0(Y_{n-k,n})}\right) =$$

$$\frac{1}{\hat{\gamma}_n(k_0, k)\sum\limits_{j=1}^{m} s_j^2}\sum_{j=1}^{m} s_j \log(e^{c\gamma s_j}(1+o_p(1))) =$$

$$\frac{1}{\hat{\gamma}_n(k_0, k)\sum\limits_{j=1}^{m} s_j^2}\sum_{j=1}^{m} s_j (c\gamma s_j(1+o_p(1))) \tag{8}$$

由文献[1]引理 2.1 知

$$\hat{\gamma}_n^H(k_0, k) \to \gamma \tag{9}$$

从而 $\hat{C}_n(k_0, k) \xrightarrow{P} c$.

讨论当 $x > 0$ 时 $(\gamma_n^H(k_0, k), \hat{c}_n(k_0, k)) \xrightarrow{P} (\gamma, c)$ 的渐近分布，需要二阶条件：

$$\lim_{t\to\infty}\frac{\log U_0(tx) - \log U_0(t) - \gamma \log x}{\beta_0(t)} = \frac{x^\rho - 1}{\rho} \tag{10}$$

及二阶强化条件

$$\lim_{t\to\infty}\frac{\log U_{s_j}(t) - \log U_0(t) - c\gamma s_j}{\beta_0(t)} = \frac{e^{c\rho s_j} - 1}{\rho} \tag{11}$$

其中 $\lim_{t\to\infty}\beta_0(t) = 0$，$|\beta_0(t)| \in RV_\rho$，$\rho \leqslant 0$.

引理 1 如果(10)式和(11)式成立，则

$$\lim_{t\to\infty}\frac{\log U_{s_j}(tx) - \log U_{s_j}(t) - \gamma\log x}{\beta_{s_j}(t)} = \frac{x^\rho - 1}{\rho} \tag{12}$$

其中 $\beta_{s_j}(t) := e^{c\rho s_j}\beta_0(t)$，$j = 1, 2, \cdots, m$.

证 由于 $|\beta_{s_j}(t)| \in RV_\rho$，因此 $\lim\limits_{t\to\infty}\frac{\beta_0(tx)}{\beta_0(t)} = x^\rho$. 则由(11)式可得

$$\lim_{t\to\infty}\frac{\log U_{S_j}(tx) - \log U_0(tx) - c\gamma S_j}{\beta_0(t)} \cdot \frac{\beta_0(t)}{\beta_0(tx)} = \frac{c\rho S_j - 1}{\rho}$$

$$\lim_{t\to\infty}\frac{\log U_{S_j}(tx) - \log U_0(tx) - c\gamma S_j}{\beta_0(t)} = x^\rho \frac{e^{c\rho s_j} - 1}{\rho}$$

$$\lim_{t\to\infty}\frac{\log U_{s_j}(tx) - \log U_0(tx) - (\log U_{s_j}(t) - \log U_0(t))}{\beta_0(t)} = (x^\rho - 1)\frac{e^{c\rho S_j} - 1}{\rho}$$

$$\lim_{t\to\infty}\frac{\log U_{s_j}(tx) - \log U_{s_j}(t) - \gamma\log x}{\beta_0(t)} = (x^\rho - 1)\frac{e^{c\rho S_j - 1}}{\rho} + \frac{x^\rho - 1}{\rho}$$

$$\lim_{t\to\infty}\frac{\log U_{s_j}(tx) - \log U_{S_j}(t) - \gamma\log x}{\beta_0(t)e^{c\rho s_j}} = \frac{x^\rho - 1}{\rho}$$

定义 $\beta_{s_j}(t) = \beta_0(t)e^{c\rho s_j}$，则结论得证.

定理 2　若 $k=k(n)$，$k_0=k_0(k)$ 满足(5)式，$U_{s_j}=U_{s_j}(t):=\left(\dfrac{1}{1-F_{s_j}}\right)^{\leftarrow}$ 满足二阶正规变化条件(12)式及二阶强化条件(11)式，则

$$\hat{c}_n(k_0,k)=c-\frac{cP_n}{\sqrt{k_0}}(1+o_p(1))-d_3\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{\rho}(1+o_p(1)) \tag{13}$$

其中 P_n 服从渐近标准正态分布，且 $d_3=\dfrac{c}{1+\gamma}$，$d_4=\dfrac{c}{(1-\rho)\gamma}\dfrac{1}{m}\sum_{j=1}^{m}\mathrm{e}^{c\rho s_j}-\dfrac{1}{\gamma\sum_{j=1}^{m}s_j^2}\sum_{j=1}^{m}s_j\dfrac{\mathrm{e}^{c\rho s_j}-1}{\rho}$.

证　由(5)式可得

$$\hat{c}_n(k_0,k):=\frac{1}{\hat{\gamma}_n(k_0,k)\sum_{j=1}^{m}s_j^2}\sum_{j=1}^{m}s_j\log\left(\frac{X_{n-k_0,n}(s_j)-X_{n-k,n}(s_j)}{X_{n-k_0,n}(0)-X_{n-k,n}(0)}\right)$$

为了便于计算我们单独考虑 $\log\dfrac{X_{n-k_0,n}(s_j)-X_{n-k,n}(s_j)}{X_{n-k_0,n}(0)-X_{n-k,n}(0)}$，由(7)式可得

$$\log\frac{X_{n-k_0,n}(s_j)-X_{n-k,n}(s_j)}{X_{n-k_0,n}(0)-X_{n-k,n}(0)}\overset{d}{=}$$

$$\log\frac{U_{s_j}Y_{n-k_0,n}-U_{s_j}X_{n-k,n}}{U_0X_{n-k_0,n}-U_0X_{n-k,n}}=$$

$$\log\frac{\left[\left(\dfrac{U_{s_j}(Y_{n-k_0,n})}{U_{s_j}(Y_{n-k,n})}-1\right)\dfrac{U_{s_j}Y_{n-k,n}}{U_0X_{n-k,n}}\right]}{\left(\dfrac{U_0Y_{n-k_0,n}}{U_0Y_{n-k,n}}-1\right)}=$$

$$\log\frac{\left[\left|\dfrac{U_{s_j}\left(\dfrac{Y_{n-k_0,n}}{Y_{n-k,n}}Y_{n-k,n}\right)}{U_{s_j}(Y_{n-k,n})}-1\right|\left(\mathrm{e}^{c\gamma s_j}\ \dfrac{\mathrm{e}^{c\rho S_j}-1}{\rho}\beta_0(Y_{n-k,n})(1+o_p(1))+\mathrm{e}^{c\gamma s_j}\right)\right]}{\left|\dfrac{U_0\left(\dfrac{Y_{n-k_0,n}}{Y_{n-k,n}}Y_{n-k,n}\right)}{U_0(Y_{n-k,n})}-1\right|}=$$

$$\log\left[\left(\frac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\gamma}-1+\beta_{s_j}Y_{n-k,n}\left(\frac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\gamma}\frac{\left(\dfrac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\rho}-1}{\rho}(1+o_p(1))\right]-$$

$$\log\left[\left(\frac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\gamma}-1+\beta_{s_0}Y_{n-k,n}\left(\frac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\gamma}\frac{\left(\dfrac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\rho}-1}{\rho}(1+o_p(1))\right]+$$

$$\log\left[\mathrm{e}^{c\gamma s_j}\ \frac{\mathrm{e}^{c\rho S_j}-1}{\rho}\beta_0(Y_{n-k,n})(1+o_p(1))+\mathrm{e}^{c\gamma s_j}\right]=$$

$$\frac{1}{\rho}\left[\left(\frac{Y_{n-k_0,n}}{Y_{n-k,n}}\right)^{\rho}-1\right](\beta_{s_j}Y_{n-k,n}-\beta_{s_0}Y_{n-k,n})(1+o_p(1))+$$

$$\log\left[\mathrm{e}^{c\gamma s_j}\ \frac{\mathrm{e}^{c\rho s_j}-1}{\rho}\beta_0(Y_{n-k,n})(1+o_p(1))+\mathrm{e}^{c\gamma s_j}\right]$$

注意到 $\beta_{s_j}(t)=\mathrm{e}^{c\rho s_j}\beta_{s_0}(t)$，并且 $\dfrac{k}{n}Y_{n-k,n}\xrightarrow{p}1$，$\dfrac{k_0}{n}Y_{n-k_0,n}\xrightarrow{p}1$，则

$$\log\frac{X_{n-k_0,n}(s_j)-X_{n-k,n}(s_j)}{X_{n-k_0,n}(0)-X_{n-k,n}(0)}=$$

$$\frac{1}{\rho}\left[\left(\frac{k_0}{k}\right)^{-\rho}-1\right](\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))+c\gamma s_j+\frac{1}{\rho}(\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))=$$

$$\frac{1}{\rho}\left(\frac{k_0}{k}\right)^{\rho}(\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))+c\gamma s_j$$

故

$$\hat{c}_n(k_0,k)=\frac{\sum_{j=1}^{m}c\gamma s_j^2+\sum_{j=1}^{m}s_j\frac{1}{\rho}\left(\frac{k_0}{k}\right)^{-\rho}(\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))}{\hat{\gamma}_n^H(k_0,k)\sum_{j=1}^{m}s_j^2}=$$

$$c\frac{\gamma}{\hat{\gamma}_n^H(k_0,k)}+\frac{1}{\hat{\gamma}_n^H(k_0,k)\sum_{j=1}^{m}s_j^2}\sum_{j=1}^{m}s_j\frac{1}{\rho}\left(\frac{k_0}{k}\right)^{-\rho}(\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))=$$

$$c+\left(\frac{\gamma-\hat{\gamma}_n^H(k_0,k)}{\hat{\gamma}_n^H(k_0,k)}\right)c+\frac{1}{\hat{\gamma}_n^H(k_0,k)\sum_{j=1}^{m}s_j^2}\sum_{j=1}^{m}s_j\frac{1}{\rho}\left(\frac{k_0}{k}\right)^{-\rho}(\mathrm{e}^{c\rho s_j}-1)\beta_0\left(\frac{n}{k}\right)(1+o_p(1))$$

由于 γ 与 s_j 无关，我们使用线性均值估计量

$$\frac{1}{m}\sum_{j=1}^{m}\hat{\gamma}_n^H(k_0,k)(s_j)=\gamma+\gamma\frac{P_n}{\sqrt{k_0}}+d_1\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))+d_2\left(\frac{k_0}{k}\right)^{-\rho}\beta_0\left(\frac{n}{k}\right)(1+o_p(1))\tag{14}$$

来代表 $\hat{\gamma}_n^H(k_0,k)$，其中 $d_1=\frac{\gamma}{1+\gamma}$，$d_2=\frac{1}{1-\rho}\frac{1}{m}\sum_{j=1}^{m}\mathrm{e}^{c\rho s_j}$(参见文献[1]定理 2.1)．因此

$$\hat{c}_n(k_0,k)=c-\frac{1}{\hat{\gamma}_n^H(k_0,k)}\left[c\frac{P_n}{\sqrt{k_0}}\gamma-\frac{c}{1+\gamma}\gamma\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-\right.$$

$$\left.\left(\frac{c}{1-\rho}\frac{1}{m}\sum_{j=1}^{m}\mathrm{e}^{c\rho s_j}-\frac{1}{\sum_{j=1}^{m}s_j^2}\sum_{j=1}^{m}s_j\frac{\mathrm{e}^{c\rho s_j}-1}{\rho}\right)\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))\right]=$$

$$c-\frac{1}{\gamma}\left[c\frac{P_n}{\sqrt{k_0}}\gamma-d_3\gamma\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))\right]\cdot\frac{\hat{\gamma}^{H_n}(k_0,k)}{\gamma}=$$

$$c-\frac{1}{\gamma}\left[c\frac{P_n}{\sqrt{k_0}}\gamma-d_3\gamma\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))\right]\cdot$$

$$\left[1-\left(\gamma\frac{P_n}{\sqrt{k_0}}+d_1\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))+d_2\left(\frac{k_0}{k}\right)^{-\rho}\beta_0\left(\frac{n}{k}\right)(1+o_p(1))\right)\right]=$$

$$c-c\frac{P_n}{\sqrt{k_0}}-d_3\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))+$$

$$\left(\frac{d_2c}{\gamma}+d_4\right)\frac{P_n}{\sqrt{k_0}}\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))+\left(\frac{(d_1+d_2)d_4}{\gamma}\right)\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{\gamma-\rho}(1+o_p(1))+$$

$$\frac{d_1d_3}{\gamma}\left(\frac{k_0}{k}\right)^{2\gamma}(1+o_p(1))+\frac{d_2d_4}{\gamma}\beta_0^2\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-2\rho}(1+o_p(1))=$$

$$c-c\frac{P_n}{\sqrt{k_0}}(1+o_p(1))-d_3\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))-d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{-\rho}(1+o_p(1))$$

则结论得证.

定理 3 若(13)式和(14)式成立，且 $k_0^{\frac{1}{2}+\gamma}k^{-\gamma}\to\lambda_1$，$k_0^{\frac{1}{2}-\rho}k^{\rho}\beta_0\left(\frac{n}{k}\right)\to\lambda_2$，则

$$\sqrt{k_0}(\hat{\gamma}_n^H(k_0,k)-\gamma,\hat{c}_n(k_0,k)-c)\xrightarrow{P}N\left(\begin{pmatrix}\mu_1\\\mu_2\end{pmatrix},\sum\right)$$

其中

$$\begin{pmatrix}\mu_1\\\mu_2\end{pmatrix}=\begin{pmatrix}d_1\lambda_1+d_2\lambda_2\\-d_3\lambda_1-d_4\lambda_2\end{pmatrix},\ \sum=\begin{pmatrix}\gamma^2&-\gamma c\\-\gamma c&c^2\end{pmatrix}$$

证 在 $k_0^{\frac{1}{2}+\gamma}k^{-\gamma}\xrightarrow{p}\lambda_1$，$k_0^{\frac{1}{2}-\rho}k^{\rho}\beta_0\left(\frac{n}{k}\right)\xrightarrow{p}\lambda_2$ 的条件下，由(13)式可得

$$\sqrt{k_0}(\hat{c}_n(k_0,k)-c)=-\left(cP_n(1+o_p(1))+d_3\left(\frac{k_0}{k}\right)^{\gamma}(1+o_p(1))+d_4\beta_0\left(\frac{n}{k}\right)\left(\frac{k_0}{k}\right)^{\rho}(1+o_p(1))\right)\to$$

$$-(cP_n+d_3\lambda_1+d_4\lambda_2)$$

同理可得

$$\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma)\rightarrow\gamma(P_n+d_1\lambda_1+d_2\lambda_2)$$

对于任意的非零实数 l_1 和 l_2，

$$l_1\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma)+l_2\sqrt{k_0}(\hat{c}_n(k_0, k)-c)\rightarrow(l_1\gamma-l_2c)P_n+(l_1d_1-l_2d_3)\lambda_1+(l_1d_2+l_2d_4)\lambda_2\rightarrow N(\mu, \sigma^2)$$

其中：$\mu=(l_1d_1-l_2d_3)\lambda_1+(l_1d_2+l2d_4)\lambda_2$，$\sigma^2=(l_1\gamma-l_2c)^2$.

由多元正态分布的性质可知，$\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma$，$\hat{c}_n(k_0, k)-c)$ 服从二元正态分布 $N\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}, \sum\right)$，其中

$$E(\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma))=E(\gamma(P_n+d_1\lambda_1+d_2\lambda_2))\rightarrow d_1\lambda_1+d_2\lambda_2:=\mu_1$$

$$E(\hat{c}_n(k_0, k)-c)=E(-(cP_n+d_3\lambda_1+d_4\lambda_2))\rightarrow-(d_3\lambda_1+d_4\lambda_2):=\mu_2$$

$$Var(\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma))=Var(\gamma(P_n+d_1\lambda_1+d_2\lambda_2))\rightarrow\gamma^2$$

$$Var(\sqrt{k_0}(\hat{c}_n(k_0, k)-c))=Var(-(cP_n+d_3\lambda_1+d_4\lambda_2))\rightarrow c^2$$

$$\begin{aligned}&2Cov(\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma, \hat{c}_n(k_0, k)-c))=\\&Var(\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma+\hat{c}_n(k_0, k)-c))-\\&Var(\sqrt{k_0}(\hat{\gamma}_n^H(k_0, k)-\gamma))-Var((\sqrt{k_0}\hat{c}_n(k_0, k)-c))\rightarrow-2\gamma c\end{aligned}$$

则结论得证.

参考文献：

[1] FRAGA A M I. A Location Invariant Hill-Type Estimator [J]. Extremes，2001，4(3)：199－217.

[2] DE HAAN L，TANK A K，NEVES C. On Tail Trend Detection：Modeling Relative Risk [J]. Extremes，2015，18(2)：141－178.

[3] PICKANDS J. Statistical Inference Using Extreme Order Statistics [J]. Annals of Statistics，1975，3(1)：119－131.

[4] CHENG S，HAAN L D. Penultimate Approximation for Hill's Estimator [J]. Scandinavian Journal of Statistics，2001，28(3)：569－575.

[5] DE HAAN L，FERREIRA A. Extreme Value Theory [M]. Berlin：Springer，2006.

[6] 凌成秀. 位置不变的矩型估计量的渐近性质 [D]. 重庆：西南师范大学，2005.

On Estimation of Heavy Tail Parameters at Different Observation Points

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Abstract：Based on the asymptotic properties of Hell-type estimators with position invariance，a position invariant estimator $\hat{c}_n(k_0, k)$ for different observation points of extreme rainfall has been presented in this paper，and its weak consistency and asymptotic normal expansion of its distribution also been discussed.

Key words：tail distribution；extreme value index；location invariant；hill estimation

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